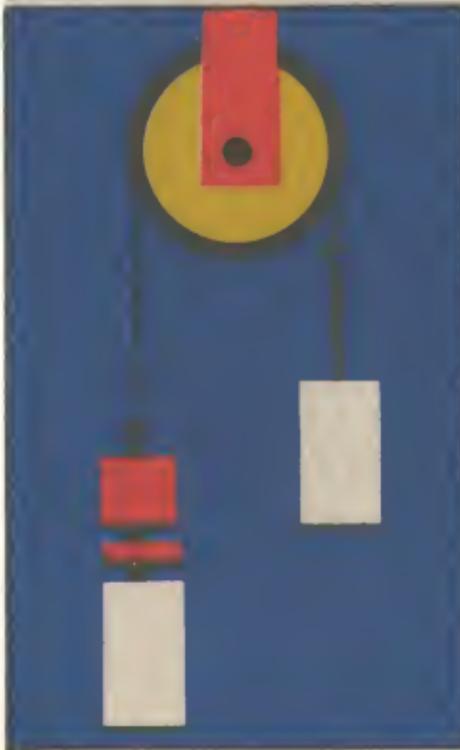


Fundamentals of Kinematics. Nonuniform Rectilinear and Curvilinear Motion. Fundamentals of Dynamics. Laws of Motion. Forces in Nature. Fundamentals of Statics. Conservation Laws in Mechanics. Laboratory Works.

Senior

# PHYSICS 1

I. K. Kikoin, A. K. Kikoin



Mir Publishers Moscow

metre

m

Paris



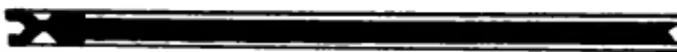
1 m is approximately equal to  $1/40\,000\,000$  of the Earth meridian passing through Paris

kilogram

kg



1 kg  
approximately  
to the mass  
of pure  
water  
at a temperatu

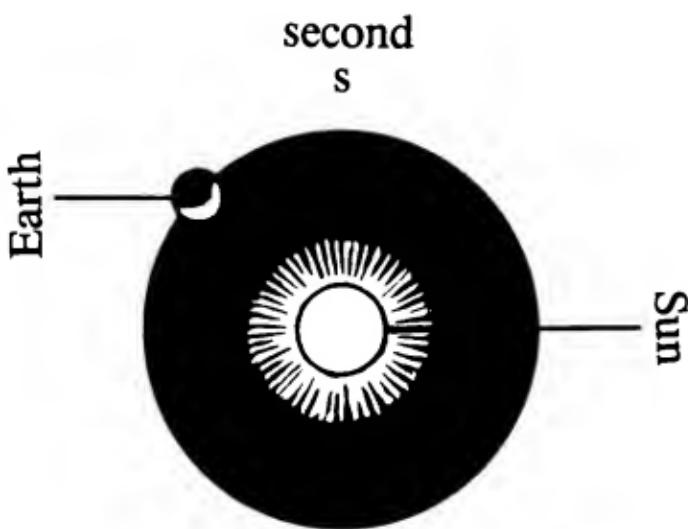


standard metre

1 m is equal to 1 650 763.73 of the wavelength corresponding to the orange line emitted by a crypton atom having an atomic mass of 86



standard kilogram



1 s is approximately  
equal to  $1/31\ 556\ 925.9747$   
of the period of revolution  
of the Earth  
about the Sun in 1900

1 s is equal  
to 9 192 631 770 periods  
of radiation corresponding to  
a transition between certain  
two levels of a caesium atom  
having an atomic mass of 133



Senior

# PHYSICS

1



И К Кикони  
А К Кикони  
Физика  
Учебник для  
8 класса  
Москва  
"Просвещение"

I. K. Kikoin, A. K. Kikoin

Senior

# PHYSICS 1



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# 0 MECHANICS

## Introduction

Everything that really exists in the world, on the Earth or out of it, is called *matter*. Various bodies surrounding us and the substances of which these bodies are composed are material. Sound, light, radiowaves are also material objects (although they cannot be called bodies) since they actually exist. This means that any object (and, in general, the material world surrounding us) exists independently of our consciousness and acts (or may act) on our organs of sense.

One of the basic properties of matter is its variability. Various changes occurring in the material world, viz. changes in matter, are called *natural phenomena*.

Physics is a science dealing with unanimate nature. It studies the properties of matter, its various changes, the laws describing these changes and the relationships between different phenomena.

A distinctive feature of physics in comparison with other sciences is that while studying the properties of matter and its changes, various physical quantities are introduced. These quantities can be measured and expressed by figures. This makes it possible to express the course of phenomena and their interrelations with the help of mathematical relations between the introduced physical quantities. The most important relations existing for natural phenomena, which are called *laws*, are also represented in the form of mathematical formulas.

The role of mathematics in physics is clearly outlined by the famous Italian scientist Galileo: "Philosophy<sup>11</sup> is written in a magnificent book that is always open before our eyes (I mean the Universe), but which cannot be understood without learning first its language and without recognizing the characters used for its writing. Its language is the language of mathematics and the characters are triangles and other geometrical figures without which not a single word can be understood. Without them, we can only wander blindly over the pitch-dark labyrinth".

<sup>11</sup> In Galileo's time, philosophy meant physics.

Far from all properties of matter and the laws of nature are known. However, the evolution of physics and other sciences shows that there is nothing that cannot be studied, learnt and understood. The cognizability of the material world can also be considered as one of its important properties.

The knowledge of the properties of matter and the laws of its variation (the laws of nature) is in cope with the natural tendency of a human being to know and understand the world around him. Therefore, this knowledge constitutes an important part of human culture. On the other hand, natural sciences are of utmost practical importance, since they enable us to predict the course of various phenomena and processes. And without it no enterprize can exist! For example, an engineer knows how a machine will operate even before it is constructed, since while designing it, he used information delivered by science, and above all, physics. The knowledge of the laws of nature makes it possible not only to predict the future but also to explain the past, since the laws of nature have been the same in the past and will remain unchanged forever.

The possibility to forecast the future on the basis of laws of nature has become especially important nowadays when the activity of human being, who mastered a powerful technique, strongly affects the environment. In order to avoid an irretrievable disaster due to this influence of human activity on nature, people must be able to foresee in advance the possible aftereffects. For this the deeper and deeper knowledge of the laws of nature, including those studied in physics, is required.

Mechanical motion is the best studied among all phenomena in nature. The branch of physics in which this phenomenon is investigated is called *mechanics*. Our book is devoted to this part of physics.

# Fundamentals of Kinematics

## 1

### GENERAL CONCEPTS OF MOTION

#### BASIC PROBLEM OF MECHANICS

All events occur somewhere in space (where?) and at a certain instant of time (when?). In particular, at any moment a body occupies a certain position relative to other bodies in space. If the position of the body in space varies with the passage of time, we say that the body moves, or is in mechanical motion.

*The mechanical motion of a body is defined as the change of its position in space relative to other bodies with the passage of time.*

To study the motion of a body means to determine how its position changes with time. If this is known, we can determine (calculate) the position of the body at any instant of time. This is the essence of the *basic problem of mechanics*, i.e. the problem of *finding the position of the body at any moment*. Thus, astronomers use the laws of mechanics to determine the relative positions of celestial bodies and quite accurately predict celestial phenomena like solar and lunar eclipses. What is more, if the historians did not exactly know, say, the exact date of Prince Igor's march on the Polovets tribes, the astronomers could come to their assistance. In the famous Russian epic "The Lay of Igor's Host" written in praise of this campaign it is mentioned that Igor's march on the Polovets soil coincided with a total solar eclipse. This information is sufficient to establish that Igor's forces were on the border of the Polovets territory on May 1, 1185.<sup>11</sup>

Bodies can be in quite diverse types of mechanical motion: they can move along different trajectories, faster or slower, etc. To solve the basic problem of mechanics, we must be able to describe briefly and exactly the motion of the body, i.e. indicate how its position changes with time. In other words, we must find the mathematical *description* of the motion, viz. establish the connection between the quantities characterizing the mechanical motion. In the first part of mechanics, called *kinematics*, we shall consider these quantities and establish the relation between them.

<sup>11</sup> There is no chance of an error in this case, since it is well known that a total solar eclipse is observed at the same place once in about 200 years. Only one solar eclipse could be observed in the steppes of the Don in the 12th century.

## 1.1.

### Translational Motion of Bodies. Material Point

To study the motion of a body, i.e. the change of its position in space, we must first of all know how to determine this position. But here we encounter certain difficulty. Each body has a definite size. Consequently, its different parts have different positions in space. How, then, can we determine the position of the body? In the general case, it is not an easy task. However, in many cases it is not necessary to indicate the position of each point of a moving body.

The position of each individual point of the body need not be specified if all the points move identically.

For example, there is no need to describe the motion of each point of a sleigh which a boy pushes uphill since each point moves in exactly the same way as all the other points.

All the points of a barge floating in a river move identically, and so do the points of a suitcase as we lift it from the floor (Fig. 1).

*The motion of a body in which all its points move identically is called translational motion.* Any imaginary straight line drawn in a body performing translational motion remains parallel to itself.

*The motion of each point of a body also need not be specified when the size of the body is small in comparison with the distance traversed by it or in comparison with the distance from the body to other bodies.* In these cases, the size of the body can be neglected. For example, an ocean liner is small in comparison with stretch of its voyage, and hence we can treat the liner as a point while describing its motion in the ocean.

The same approach is used by astronomers while studying the motion of celestial bodies. Planets, stars and the Sun are by no means small objects. However, the radius of the Earth, for example, is about 1/24 000 of the distance between the Earth and the Sun. Hence we can assume that the Earth is a point which moves around another point, viz. the centre of the Sun.

Speaking of the motion of a body, we shall actually mean the motion of some point of this body. It should be borne in mind that this is a material point which differs from ordinary bodies in that it has no dimensions.

*A material point is a body whose size can be neglected under the given conditions of motion.*

The expression "under the given conditions" means that the same object can be treated as a material point for some type of motion while this may not be possible for other motions. For example, suppose that a boy going to school covers the distance of one kilometre. He can be treated as a material point in this motion, because he is small in comparison with the distance he



Fig. 1

travels. However, when the same boy does his morning exercises, he can by no means be treated as a material point.

---

?

In which of the following cases can the bodies be treated as material points?

1. A discus is being machined on a lathe. The same discus covers 55 m when thrown by an athlete.
2. An aeroplane flies from Moscow to Khabarovsk. A plane spins round its axis in aerobatic manoeuvres.
3. A skater covers a distance during a competition. A skate-dancer does free skating exercises.
4. The flight of a spaceship is watched from the mission control centre on the Earth. The same spaceship is watched by a cosmonaut carrying out docking manoeuvres with this ship in space.

---

## 1.2.

### Position of a Body in Space. Reference System

How is the position of a body determined? An ancient manuscript pertaining to the beginning of our era gives the following description of the location of a treasure: "Stand at the East corner of the last house of the village and take 120 steps northwards. Then face the East and take 200 steps. At this spot, dig a hole 10 cubits deep, and find 100 talents of gold". If the village and the house mentioned in the document existed today, we could easily find this treasure. However, for obvious reasons, there is no trace left of the house and the village, and hence it is impossible to find the treasure. This example shows that the position of a body or a point can be specified only *relative to* some other body which is called the *reference body*.

The reference body can be chosen quite arbitrarily. For example, it can be the house in which we live, the coach of the train by which we are travelling or any other body. The Earth, the Sun and stars can also serve as reference bodies.

**COORDINATES OF A POINT.** When a reference body has been chosen, the coordinate axes are drawn through some of its points, and the position of any point of a body is defined by its coordinates. This procedure is well known from the course of elementary mathematics.

Let us determine, for example, the position of two motorcars *I* and *II* on a road (Fig. 2). We draw the coordinate axis *OX* along the road and fix the reference point (origin of coordinates) at point *O*. We assume that positive coordinates are measured to the right of point *O* and negative, to the left of it. Then the position of motorcar *I* is determined by its coordinate  $x_I = OB$ . The scale in Fig. 2 is chosen in such a way that  $x_I = 1200$  m. The coordinate of motorcar *II* is expressed by the number 400 m, but since it has to be measured to the left of the reference point, we write  $x_{II} = -400$  m. Thus,

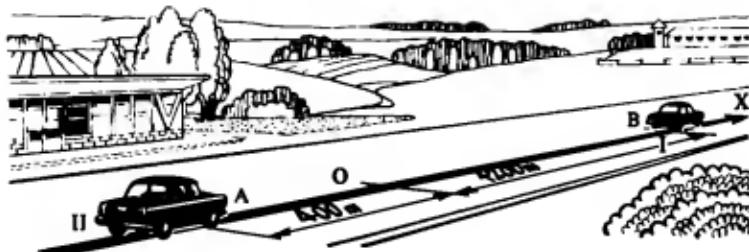


Fig. 2

the position of a body on a straight line is defined by a single coordinate.

If a body can move in a plane (for example, a boat on a lake), two coordinate axes  $OX$  and  $OY$  are drawn through points chosen on the reference body. The position of a body on a plane is defined by two coordinates  $x$  and  $y$ . For example, point  $A$  (Fig. 3) has the coordinates  $x = 3$ ,  $y = 4$ , while the coordinates of point  $B$  are  $x = 2$ ,  $y = -1.5$ .

Finally, to specify the position of a body in space (for example, an aeroplane in flight), we must draw three mutually perpendicular coordinate axes  $OX$ ,  $OY$  and  $OZ$  through the reference body (Fig. 4). The position of a body (point) in space is accordingly defined by three coordinates  $x$ ,  $y$  and  $z$ .

This is the system of coordinates that was chosen to describe the location of the treasure in the document mentioned at the beginning of this section. To find the treasure, we must only know the location of the reference body.

Thus, the position of a point on a line, in a plane or in space is defined respectively by one, two or three coordinate numbers. The space surrounding us is, so to say, a space of three dimensions or a three-dimensional space.

**REFERENCE SYSTEM.** It was mentioned above that mechanics deals with determining the position of a body at any instant of time. Since the position of a body is defined by the coordinates of its points, the basic problem of mechanics is reduced to determining the coordinates of points of a body at any instant of time.

The system of coordinates, the reference body to which it is fixed, and the instrument for measuring time form together the reference system with respect to which the motion of a body is considered.

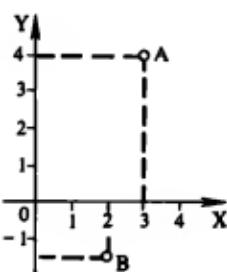


Fig. 3

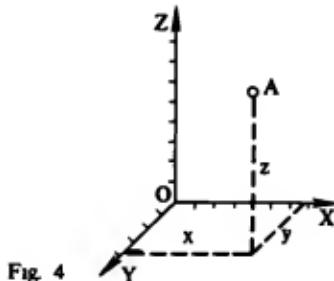


Fig. 4

As a body moves, the coordinates of its points change. If, for example, the coordinates of a point along  $X$ -,  $Y$ - and  $Z$ -axes are equal to  $x_0$ ,  $y_0$ ,  $z_0$  at some instant  $t = 0$ , and become equal to  $x$ ,  $y$ ,  $z$  after a certain time interval  $t$ , this means that the  $x$ -,  $y$ - and  $z$ -coordinates have changed during this interval of time by  $x - x_0$ ,  $y - y_0$  and  $z - z_0$  respectively. These three quantities are the *variations* in the  $x$ -,  $y$ - and  $z$ -coordinates respectively. The variation of a quantity is sometimes denoted by the symbol  $\Delta$  (Greek "delta"), for example,  $x - x_0 = \Delta x$ ,  $y - y_0 = \Delta y$ ,  $z - z_0 = \Delta z$ .

## 1.3. Displacement

The change in coordinates is associated with the first of the quantities mentioned in Introduction which were introduced for describing motion of a body, viz. *displacement*. What is displacement?

Suppose that at some initial instant, a moving body (point) occupies a position  $M_1$  (Fig. 5). After a certain interval of time, the body occupies a different position at a distance  $s$  from the initial point. How to find the new position of the body? Obviously, it is not enough to know the distance  $s$  for this purpose, since there is an infinite number of points at a distance  $s$  from point  $M_1$  (see Fig. 5).

When a body moves, its motion takes place in a certain direction. To find the new position of the body, we must know the direction of the line segment connecting the initial and final positions of the body. This directed segment describes the displacement of the body. The tip of the segment is marked by an arrow for the sake of clarity. Setting this segment to point  $M_1$ , we find the new position  $M_2$  of the body at the tip of the arrow (Fig. 6).

*The displacement of a body (material point) is the directed line segment connecting the initial position of the body to its final position.*

The displacement of a body must be distinguished from its trajectory (the path along which it moves). The fact that the body has moved from point  $M_1$  to point  $M_2$  (Fig. 7) and its displacement is equal to the length of segment  $M_1M_2$  does not mean that the body was moving along the straight

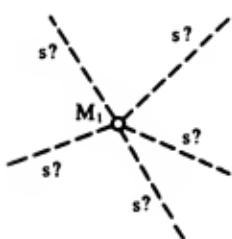


Fig. 5

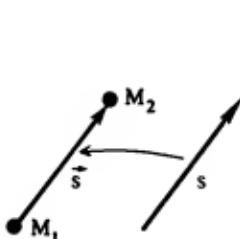


Fig. 6

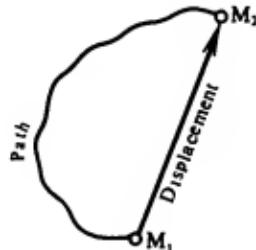


Fig. 7

line  $M_1M_2$ . The trajectory of motion of the body, i.e. the line along which it actually moved, may not coincide with this straight line. The following example will explain this point.

Figure 8 shows the map of the Black Sea region. The distance between Odessa and Sevastopol' is 270 km as the crow flies. In order to reach Sevastopol' from Odessa, we require a displacement of 270 km in the southeast direction. If we take a cruise on a steamer, the actual course may indeed coincide with the straight line representing this displacement. However, we can also take a train from Odessa to Sevastopol'. The railway line passes through Nikolaev, Kherson and Dzhankoi. The length of the track is 660 km. If we travel by train, the trajectory of motion no longer coincides with the displacement.

Thus, in order to find the position of a body at any instant of time, we must know its initial position and displacement to this moment.

?

1. Observation of the movements of players in a football match showed that the forward covered about 12 km during the match. How should this distance be interpreted: as the displacement or path length?
2. A navigator tracking the position of his ship in the morning finds that the ship is at a distance of 100 km from the point where it was the previous evening. What does this number represent: the displacement or path traversed by the ship?
3. Taking over the motor-car from a taxi-driver at the end of a day, the man on duty at the garage recorded that the meter reading had gone up by 300 km. What does this reading indicate: the distance covered by the car or the displacement?

## 1.4. On Vector Quantities

The quantity "displacement" differs from many other quantities in that we must know its magnitude as well as the direction in which it has occurred. Quantities like displacement, which are specified not only by their magnitude (modulus) but also by direction, are called *vector quantities*. A vector quantity is represented by a segment which starts at a certain point and terminates at an arrow-tip indicating its direction (see Fig. 6). Such an arrow-segment is called a *vector*. Sometimes, a vector quantity itself is called a vector; for example, it is said that displacement is a vector. The length of the segment on a chosen scale represents the modulus of the vector quantity. Vectors are denoted by letters with bars above them or by the bold face. For example, the displacement vector (see Fig. 6) is denoted by  $\bar{S}$ . The magnitude of this vector will be denoted by the same letter, but without a bar. In the figures vectors are denoted by letters with arrows above them.



Fig. 8

The magnitude (modulus) and direction of a vector are equally important. Equal vectors have the same magnitudes and the same direction.

Quantities which cannot be assigned a direction and are completely characterized by a number (dimensional or dimensionless) are called *scalar quantities* or just *scalars*. For example, the number of desks in the classroom, or the length, width and height of a room are expressed only by numbers and are therefore scalars. The magnitude of a vector is also a scalar quantity.

**OPERATIONS ON VECTORS.** The most important operation for us is the *addition of vectors*.

Suppose that  $\vec{a}$  (Fig. 9) is the displacement vector for a group of tourists moving eastwards. Undergoing this displacement, the group turns in the north-east direction and continues its hike. Let  $\vec{b}$  (see Fig. 9) be the displacement vector for the group in the north-east direction. We draw a vector  $\vec{c}$  from the beginning of vector  $\vec{a}$  to the end of vector  $\vec{b}$ . Had the group undergone just one displacement  $\vec{c}$ , it would have come to the same point where it arrived after two displacements  $\vec{a}$  and  $\vec{b}$ . Hence vector  $\vec{c}$  is the sum of vectors  $\vec{a}$  and  $\vec{b}$ . It can be seen from Fig. 9 that the magnitude of vector  $\vec{c}$  is not equal to the sum of moduli of vectors  $\vec{a}$  and  $\vec{b}$ . This is so because the three vectors  $\vec{a}$ ,  $\vec{b}$  and  $\vec{c}$  form a triangle of vectors and one side of any triangle is always smaller than the sum of the other two sides. Hence it is said that *the vectors are added not algebraically but geometrically*. Thus we arrive at the following rule for addition of two vectors. *In order to add two vectors, we must arrange them in such a way that the origin (tail) of one vector coincides with the tip (head) of the other. The vector drawn from the origin of the first vector to the tip of the second vector is the sum of the two vectors.* This rule is called the *triangle rule*.

The same result can also be obtained by means of a different construction. Preserving the length and direction of both vectors  $\vec{a}$  and  $\vec{b}$  (see Fig. 9), we arrange them in such a way that they originate from the same point

(Fig. 10). Assuming that these two vectors form the adjacent sides of a parallelogram, we construct this parallelogram and draw a diagonal from the point where origins of the two vectors meet. This diagonal (with an arrow!) is the resultant vector. The derivation of a vector sum in this way is called the *parallelogram rule*. It can be seen from Figs. 9 and 10 that both these rules lead to the same result. These rules are applicable not only to displacement vectors, but also to any other vector quantities.

**SUBTRACTION OF VECTORS.** It is often necessary to subtract one vector from another. As in the case of numbers, subtraction can always be reduced to the addition. For example, the expression  $7 - 4 = 3$  can be replaced by  $7 = 4 + 3$ . Similarly, the equality  $\vec{a} - \vec{b} = \vec{c}$  can be replaced by  $\vec{a} = \vec{b} + \vec{c}$ . Consequently, to subtract one vector ( $\vec{b}$ ) from another ( $\vec{a}$ ) means to find such a vector  $\vec{c}$  whose sum with vector  $\vec{b}$  is equal to  $\vec{a}$ . This can be done with the help of the following construction. Suppose that we have to find the difference  $\vec{a} - \vec{b}$  of vectors  $\vec{a}$  and  $\vec{b}$  (Fig. 11, top). We translate vectors  $\vec{a}$  and  $\vec{b}$  so that they originate from the same point. We then connect their tips by a vector directed from the subtrahend to the minuend (from the tip of  $\vec{b}$  to that of  $\vec{a}$ ). This vector is precisely vector  $\vec{c}$  (Fig. 11, bottom). It is seen from the figure that the sum of vectors  $\vec{b}$  and  $\vec{c}$  is indeed equal to vector  $\vec{a}$ .

*In order to find the difference between two vectors, they should be arranged so that they emerge from the same point. Their tips must then be connected by a vector directed from the subtrahend to the minuend. This vector is the difference of the two vectors.*

**COLLINEAR VECTORS.** The vectors directed along the same straight line or parallel to each other are called *collinear*. They can have the same direction (Fig. 12a) or opposite directions (Fig. 13a). These vectors are summed up in the same way as vectors  $\vec{a}$  and  $\vec{b}$  in Fig. 9: the origin of the second vector is made to coincide with the tip of the first vector (Figs. 12b and 13b), and the resultant vector is directed from the origin of the first vector to the tip of the second one. It can be seen from the figures, however, that there is no need to resort to geometric constructions. The resultant vector is modulo equal to the arithmetic sum (Fig. 12b) or difference (Fig. 13b) of the magnitudes of the vectors being added. It is directed either in the same way as the two vectors or as the vector which has the larger magnitude.

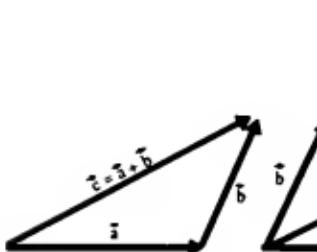


Fig. 9

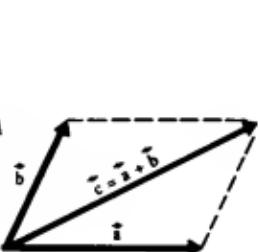


Fig. 10

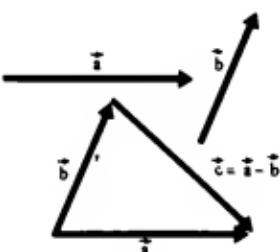


Fig. 11

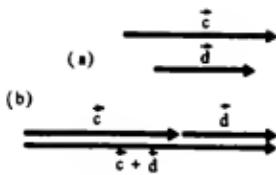


Fig. 12

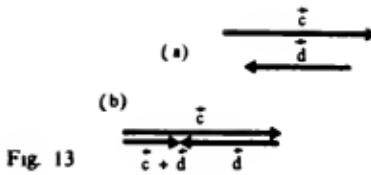


Fig. 13

**MULTIPLICATION OF A VECTOR BY A SCALAR.** In many cases, a vector  $\vec{a}$  has to be multiplied by a number  $k$  (which may be a dimensional quantity). As a result, we obtain a new vector  $k\vec{a}$ . Its direction is the same as that of vector  $\vec{a}$  if  $k > 0$  and opposite to that when  $k < 0$ . The magnitude of the new vector is equal to the product of the magnitude of vector  $\vec{a}$  and the absolute value of the number  $k$ .

?

1. What is the difference between a scalar and a vector quantity?
2. Is the quantity measured by the distance meter of a motorcar scalar or vector?
3. Two vectors are equal in magnitude but have opposite directions. Can we say that these vectors are equal to each other?
4. In which case is the magnitude of the sum of two vectors equal to the sum of their magnitudes?
5. When is the magnitude of the sum of two vectors equal to the difference of their magnitudes?

#### Hometask

1. Prove by construction that the sum of two vectors is independent of the order of summands, i.e.  $\vec{a} + \vec{b} = \vec{b} + \vec{a}$ .
2. Take two arbitrary (noncollinear) vectors and find their sum with the help of the parallelogram rule. Using the same drawing, find the difference of these vectors.
3. Find by construction the sum and difference of two vectors having the same magnitude and perpendicular to each other.

## 1.5.

### Projections of a Vector onto Coordinate Axes and Operations on Projections

It was mentioned in Sec. 1.3 that if we "attach" the displacement vector to the initial position of a moving body, the subsequent position of this body will be determined by the tip of this vector. In actual practice, however, it is impossible to "attach" vectors in this way, and the final position of a body cannot be determined by this method.

The final position of a body, i.e. its coordinates, must be calculated with the help of the displacement vector. However, it is not possible to use vectors in calculations since a vector is characterized not only by its magnitude but also by the direction.

**PROJECTIONS OF A VECTOR ONTO COORDINATE AXES.** In order to calculate the coordinate, we shall use another important concept, viz. the *projections* of a vector onto coordinate axes.

Figure 14 shows the coordinate axis  $OX$  and a vector  $\vec{a}$  which is coplanar with this axis. We drop perpendiculars  $AA_1$  and  $BB_1$  from the endpoints  $A$  and  $B$  of vector  $\vec{a}$  onto the  $X$ -axis. The feet of the perpendiculars (points  $A_1$  and  $B_1$ ) are the *projections* of points  $A$  and  $B$  onto the  $X$ -axis. The length of segment  $A_1B_1$  between the projections of the tail and head of the vector onto the axis, taken with the plus or minus sign, is called the *projection* of the vector onto the  $X$ -axis.

A projection is assumed to be positive if we have to move along the positive direction of the axis from the projection of the tail to the projection of the head of the vector and negative in the opposite case. According to this rule, the projection of vector  $\vec{a}$  (Fig. 14) is positive, while the projection of vector  $\vec{b}$  (Fig. 15) is negative. If a vector is normal to an axis (Fig. 16), its projection onto this axis is equal to zero.

The projection of a vector onto an axis is denoted by the same letter

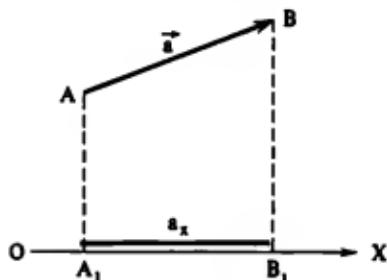


Fig. 14

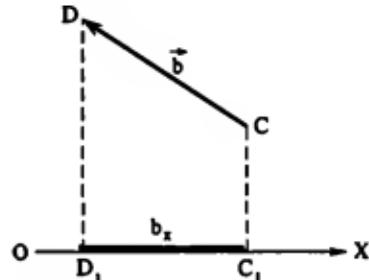


Fig. 15

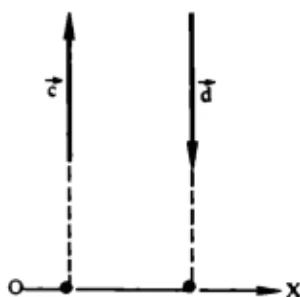


Fig. 16

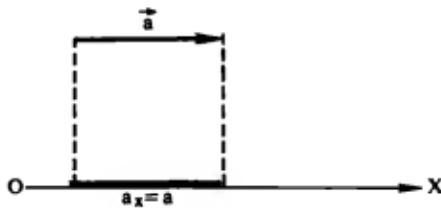


Fig. 17

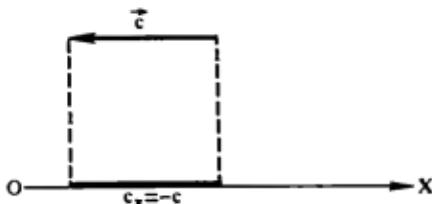


Fig. 18

which denotes the vector without an arrow and with the subscript corresponding to the axis. For example, the projections of vectors  $\vec{a}$  and  $\vec{b}$  onto the  $X$ -axis (see Figs. 14 and 15) are denoted by  $a_x$  and  $b_x$  respectively.

When a vector is parallel to an axis (Figs. 17 and 18), the magnitude of its projection onto this axis is equal to the magnitude of the vector itself. Moreover, if the vector and the axis have the same direction (see Fig. 17), the projection is positive, and if the vector and the axis have opposite directions, the projection is negative (see Fig. 18).

**PROJECTION OF THE SUM AND DIFFERENCE OF VECTORS.** Figure 19 shows vectors  $\vec{a}$  and  $\vec{b}$  and the resultant vector  $\vec{c} : \vec{c} = \vec{a} + \vec{b}$ , as well as the projections of the three vectors,  $a_x$ ,  $b_x$  and  $c_x$ , onto the  $X$ -axis. It can be seen that the projection of the resultant is equal to the sum of the projections of the vectors being added.

The projection of one of the vectors (vector  $\vec{b}$ ) may be negative (Fig. 20). However, the projection of the resultant as before, is equal to the sum of the projections of the two vectors, considering that the projection of one of the vectors is negative. *Consequently, the projection of the sum of vectors onto any axis is equal to the algebraic sum of projections of the vectors being added onto the same axis.* Since, as was mentioned above, the subtraction of vectors is reduced to their addition, this rule refers to the projection of the vector difference as well.

Thus, in order to find the projection of a vector sum or difference, there is no need to find the resultant and its projection. We can simply sum up the projections of all the vectors, taking into account their signs.

**COORDINATES OF A BODY (MATERIAL POINT) AND PROJECTIONS OF THE DISPLACEMENT VECTOR.** How can we

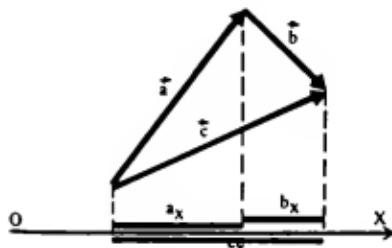


Fig. 19

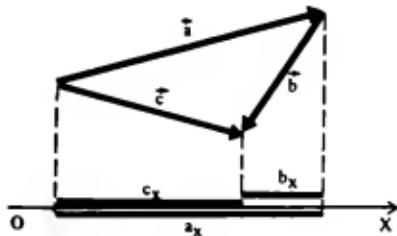


Fig. 20

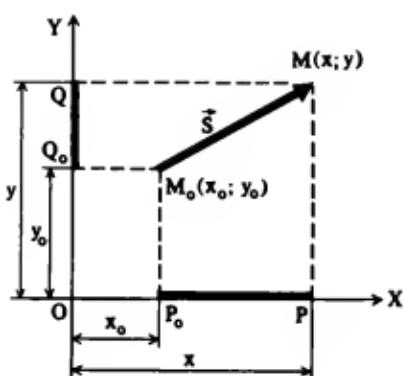


Fig. 21

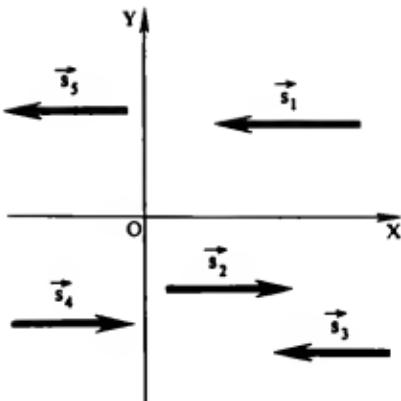


Fig. 22

determine the coordinates of the subsequent position of a body from the known coordinates of its initial position and the displacement vector?

Let us do it for a body moving in a plane.

Suppose that the displacement of the body is  $\vec{s} = \overrightarrow{M_0M}$ . We choose the coordinate system  $XOY$  so that the vector  $\overrightarrow{M_0M}$  lies in the  $XOY$ -plane (Fig. 21).

We denote the coordinates of the initial position of the body (point  $M_0$ ) by  $x_0$  and  $y_0$ , and the coordinates of a subsequent position (point  $M$ ) by  $x$  and  $y$ .

Figure 21 shows that  $OP = OP_0 + P_0P$ . But  $OP = x$ ,  $OP_0 = x_0$ ,  $P_0P = s_x$ . Consequently,

$$x = x_0 + s_x. \quad (1.5.1)$$

We can also see from this figure that  $OQ = OQ_0 + Q_0Q$ . But  $OQ = y$ ,  $OQ_0 = y_0$  and  $Q_0Q = s_y$ . Therefore,

$$y = y_0 + s_y. \quad (1.5.2)$$

Formulas (1.5.1) and (1.5.2) are valid for any other arrangement of the vector  $\overrightarrow{M_0M}$  on the  $XOY$ -plane.

Hence it follows that the projection of a displacement vector onto the  $X$ - or  $Y$ -axis is equal to the difference between the coordinates of the head and tail of this vector:

$$s_x = x - x_0, \quad s_y = y - y_0.$$

The difference between the subsequent and initial values of any quantity was called the change in this quantity. Consequently, the projection of the displacement vector  $\vec{s}$  onto the  $X$ - or  $Y$ -axis is equal to the change in the corresponding coordinate.

---

?

1. What do we call the projection of a vector onto an axis?
2. What is the relation between the displacement vector and its coordinates?
3. What is the sign of the projection of a displacement vector onto the coordinate axes if the coordinate of the moving point increases (decreases) with time?
4. What is the sign and magnitude of the projections of a displacement vector directed in parallel with one of the coordinate axes?
5. Determine the signs of the projections of the displacement vectors shown in Fig. 22 onto the  $X$ -axis. How will the coordinates of the body change upon such displacements?
6. Why is the displacement vector more important in mechanics than the distance covered by a body?
7. Can the magnitude of the displacement vector be small if the distance covered by a body is large? Give examples.

---

#### Exercise 1

1. At the initial instant, a body was at a point with coordinates  $x_0 = -2$  m and  $y_0 = 4$  m. The body moved to a point with coordinates  $x = 2$  m and  $y = 1$  m. Find the projections of the displacement vector onto the  $X$ - and  $Y$ -axes.
2. A body having coordinates  $x_0 = -3$  m and  $y_0 = 1$  m covered a certain distance after which the projection of the displacement vector onto the  $X$ -axis and  $Y$ -axis became equal to 5.2 m and 3 m respectively. Find the coordinates of the finite position of the body and draw its displacement vector. What is its magnitude?
3. A hiker covered a distance of 5 km in the southward direction and then 12 km in the eastward direction. Find the magnitude of its displacement.

---

#### Hometask

Verify the validity of formulas (1.5.1) and (1.5.2) for any position of the vector  $M_0M$  differing from that shown in Fig. 21.

---

## 1.6.

### Uniform Rectilinear Motion. Velocity

It was shown in Sec. 1.5 that for determining the coordinates of a moving body at any instant of time, we must know the projections of the displacement vector onto the coordinate axes (in other words, we must know the displacement vector). How can we find it?

Let us first consider the simplest type of motion, i.e. uniform rectilinear motion.

*Uniform rectilinear motion is a motion when a body covers equal distances in a straight line during any equal intervals of time.*

**VELOCITY.** In order to find the displacement of a body in a uniform rectilinear motion for any time interval  $t$ , we must obviously know the displacement of the body per unit time, since during any subsequent unit of time the body will cover the same distance. The displacement of a body per unit time is called its *velocity* and is denoted by letter  $v$ . The velocity of a body can be determined by measuring any path length, however small, and the time interval during which this path was traversed. If we denote the displacement along this path by  $\bar{s}$  and the corresponding time interval by  $t$ , the velocity  $\bar{v}$  will be equal to the ratio of  $\bar{s}$  to  $t$ .

*The velocity of a uniform rectilinear motion is a constant quantity equal to the ratio of the displacement of the body during any time interval to the magnitude of this interval<sup>11</sup>.*

$$\bar{v} = \frac{\bar{s}}{t}. \quad (1.6.1)$$

Since  $1/t$  is a scalar with the positive sign, velocity vector  $\bar{v}$  has the same direction as displacement  $\bar{s}$ .

If velocity  $\bar{v}$  is known, displacement  $\bar{s}$  over time  $t$  is determined from the formula

$$\bar{s} = \bar{v}t. \quad (1.6.2)$$

As was mentioned above, formulas in vector form cannot be used for calculations. For this purpose, we use formulas containing the projections of vectors onto coordinate axes instead of vectors, since algebraic operations are applicable to projections and not to vectors.

The trajectory of a body in a rectilinear motion is a straight line. Hence it is natural to direct a coordinate axis along this line. Then the motion of the body will involve the variation of only one coordinate, say, the coordinate  $x$  if the chosen axis is the  $X$ -axis. Both the velocity vector and the displacement vector are directed along this axis.

<sup>11</sup> To be more precise, the velocity of the uniform rectilinear motion is the vector whose direction is the same as that of the displacement of the body, and whose magnitude is equal to the ratio of magnitudes of the displacement and the time interval during which this displacement occurred. However, the brief but less exact definition given above, is normally used.

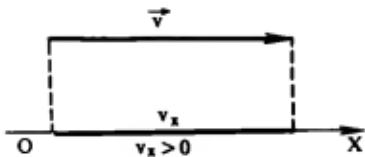


Fig. 23

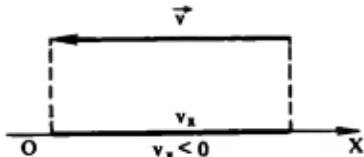


Fig. 24

**PROJECTIONS OF DISPLACEMENT AND VELOCITY.** Since the vectors  $\bar{s}$  and  $\bar{v}$  are equal, their projections on the  $X$ -axis are also equal:

$$s_x = v_x t.$$

We can now obtain a formula for calculating the  $x$ -coordinate of a point at any instant of time. We know (Sec. 1.5) that

$$x = x_0 + s_x.$$

Consequently,

$$x = x_0 + v_x t. \quad (1.6.3)$$

This formula represents the time dependence of the coordinate of a point and can therefore be used for describing the uniform rectilinear motion.

Formula (1.6.3) shows that the position of a body (material point) moving uniformly in a straight line can be determined at any instant of time if we know the initial coordinate  $x_0$  of the body (point) and the projection of the velocity vector onto the axis along which the body moves. It should be borne in mind that the projection of the velocity vector may be either positive or negative (Figs. 23 and 24).

Formula (1.6.3) clarifies the meaning of the quantity "velocity". Indeed, it follows from this formula that

$$v_x = \frac{x - x_0}{t}.$$

This means that the *projection of the velocity vector onto the axis is equal to the change in the corresponding coordinates per unit time*.

It should be emphasized once again that for solving problems in mechanics we must know not only the magnitude of the velocity but the *velocity vector* as a whole. Spidometers mounted in cars indicate just the magnitude of velocity. It is "the same" for a spidometer in which direction the car moves. Therefore, their readings cannot be used for determining the direction of motion of the car or its position at any instant of time.

---

1. What is the difference between the displacement and the distance covered by a body in uniform rectilinear motion?
2. What is the difference between the quantities defined by the expressions:  $v = s/t$  and  $\bar{v} = \bar{s}/t$ ? What do they have in common?
3. A car moves eastwards at a velocity of 40 km/hr. Another

car moves at a velocity of 40 km/hr southwards. Can we say that the cars move with equal velocities?

4. Can we find the final position of a body if we know its initial position and the length of the distance covered by it?
5. What is the relation between the velocity of a body and the change in its position during motion?

### EXAMPLES OF SOLVING PROBLEMS

1. Two motorcars are travelling towards each other at velocities 60 and 90 km/hr. At a gasoline filling station they meet and then continue on their way. Determine the position of each motorcar in 30 min after their meeting and the separation between them at this moment.

*Solution.* We take for the origin of coordinates the filling station and for the time reference point, the instant when they meet. We direct the  $X$ -axis from left to right. Then the coordinates of the motorcars in 0.5 hr after meeting can be calculated by the formulas

$$x_1 = x_{01} + v_{1x}t \quad \text{and} \quad x_2 = x_{02} + v_{2x}t.$$

The initial coordinates  $x_{01}$  and  $x_{02}$  of the two motorcars are equal to zero. Hence

$$x_1 = v_{1x}t \quad \text{and} \quad x_2 = v_{2x}t.$$

The projection  $v_{1x}$  of the velocity vector of the first motorcar is positive. This means that its velocity vector is directed in the same way as the  $X$ -axis. By hypothesis, it is equal to + 60 km/hr. The projection  $v_{2x}$  of the velocity vector of the second motorcar is negative since its velocity vector points counter to the positive direction of the  $X$ -axis, so that  $v_{2x} = - 90$  km/hr.

Consequently,

$$x_1 = 60 \text{ km/hr} \times 0.50 \text{ hr} = 30 \text{ km} \quad \text{and}$$

$$x_2 = - 90 \text{ km/hr} \times 0.5 \text{ hr} = - 45 \text{ km}.$$

The separation  $l$  between the motorcars is equal to the difference in their coordinates:

$$l = |x_2 - x_1| = |-45 \text{ km} - 30 \text{ km}| = 75 \text{ km}.$$

2. Two motorcars move along mutually perpendicular roads towards their crossing. At a certain instant of time, the first motorcar whose velocity  $v_1 = 27$  km/hr is at a distance  $l_1 = 300$  m from the crossroads. At the same instant, the second motorcar is at a distance  $l_2 = 450$  m from the crossroads. What is the speed of the second car if it reaches the crossroads in  $t = 5$  s after the first car has passed it?

*Solution.* We take the origin of coordinates at the crossroads and direct the coordinate axes  $OX$  and  $OY$  along the roads (Fig. 25). We take for the time reference point the instant when the cars were at distances  $l_1$  and  $l_2$  from the crossroads. The first car is moving along the  $X$ -axis and the second, in the negative direction of the  $Y$ -axis.

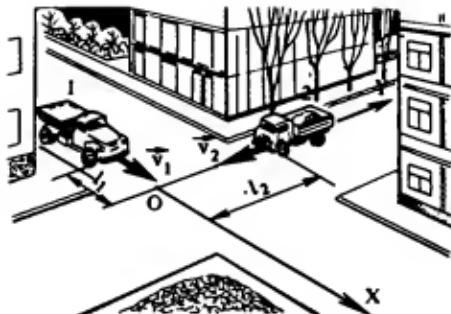


Fig. 25

During the motion of the first motorcar, only its  $x$ -coordinate varies:

$$x = x_0 + v_{1x} t,$$

while for the second car, only the  $y$ -coordinate varies:

$$y = y_0 + v_{2y} t.$$

It follows from the condition of the problem that  $x_0 = -l_1$ ,  $y_0 = l_2$ ,  $v_{1x} = v_1$ , and  $v_{2y} = -v_2$ . We denote by  $t_1$  the time when the first car passes the crossroads. At this instant, its coordinate  $x = 0$ . The second car passes the crossroads at the time  $t_1 + t$ . At this instant, its coordinate  $y = 0$ .

Consequently,

$$0 = l_1 + v_1 t_1, \quad 0 = l_2 - v_2(t_1 + t).$$

Solving these equations simultaneously, we obtain

$$v_2 = \frac{l_2}{l_1/v_1 + t} = \frac{l_2 v_1}{l_1 + t v_1}, \quad v_2 = \frac{450 \text{ m} \times 7.5 \text{ m/s}}{300 \text{ m} + 37.5 \text{ m} \cdot \text{s/s}} \\ = 10 \text{ m/s} = 36 \text{ km/hr.}$$

### Exercise 2

1. A group of hikers moving at a constant speed of 5 km/hr travels the first hour northwards, the next 0.5 hr eastwards (at right angles to the northward direction), and the last 1 hr 30 min southwards (at  $180^\circ$  to the initial direction). Where will this group be after covering these three segments? What time will the group take to return to the initial point by moving along the straight line at the same speed?
2. A car moving at a speed of 30 km/hr has covered half the distance to the destination point during a certain time. At what speed should it move to be able to reach the destination point and return back during the same span of time?
3. A traveller caught in a thunderstorm saw a flash of lightning and heard the thunder 10 s later. At what distance did the flash occur if the speed of sound in air is 340 km/hr?

## 1.7.

### Graphic Representation of Motion

**MOTION PLOT.** Motion can be visually described with the help of *plots* (graphs). If we plot on the horizontal axis (axis of abscissas) the time that has passed from the time reference point on a certain scale and on the vertical axis (axis of ordinates) the coordinate of the body on the corresponding scale, the obtained graph will express the time dependence of the coordinate of the body (it is also called the *motion plot*).

Let us suppose that a body moves uniformly along the  $X$ -axis (Fig. 26). This means that only its  $x$ -coordinate is varying. At instants  $t = 0$ ,  $t_1 = 10$  s,  $t_2 = 20$  s,  $t_3 = 30$  s, etc. the body is respectively at points whose coordinates are  $x_0 = 3$  m (point  $A$ ),  $x_1 = 4$  m,  $x_2 = 5$  m, etc.

The plot of motion is obtained if we lay the values of  $x$  along the vertical axis and the values of  $t$  along the horizontal axis (Fig. 27). The plot of this motion is a *straight line*. This means that the coordinate *linearly* depends on time.

The plot expressing the time dependence of the coordinate of a body (see Fig. 27) should not be confused with the trajectory of motion of the body, viz. the line all whose points the body visited during its motion (see Fig. 26).

For rectilinear motion, motion plots give the complete solution of the problem of mechanics since they allow us to find the position of the body at any instant of time, including the instants preceding the initial moment (if we assume that the body moved at the same speed before it reached the time reference point).

Extending the graph shown in Fig. 27 in the direction opposite to the positive direction of the time axis, we can find, for example, that 30 s before the body reached point  $A$ , it was at the zero point of the  $x$ -coordinate ( $x = 0$ ).

The form of the plots describing the time dependence of a coordinate gives the idea about the speed of motion. It is clear that the steeper the plot, i.e. the larger the angle between the plot and the time axis, the higher the speed of motion.

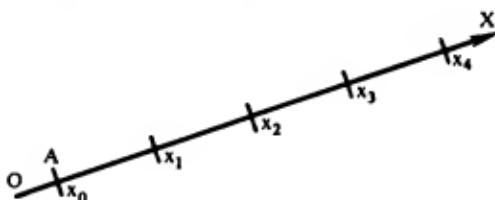


Fig. 26

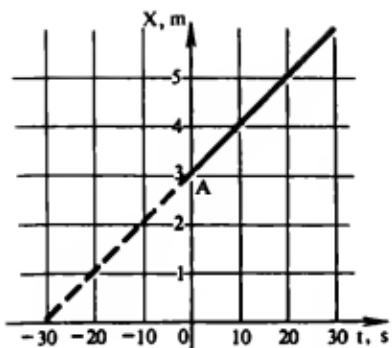


Fig. 27

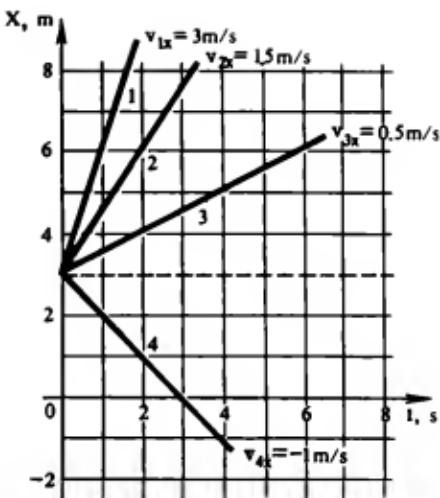


Fig. 28

Figure 28 represents several plots of motion for different velocities. Plots 1, 2 and 3 indicate that the bodies move in the positive direction of the  $X$ -axis. Straight line 4 describes the motion of a body in the direction opposite to that of the  $X$ -axis.

The plots of motion can also be used to find the displacement of a moving body for any interval of time. Figure 28 shows, for example, that the magnitude of the displacement of a body whose motion plot is marked by figure 3 during the first four seconds in the positive direction of the  $X$ -axis is equal to 2 m. During the same time, the body whose motion is described by plot 4 was displaced in the negative direction of the  $X$ -axis by 4 m.

**VELOCITY GRAPH<sup>11</sup>.** Besides motion graphs, velocity graphs are also often used. These graphs are obtained by plotting the projection of the velocity of a body along the ordinate axis, while along the abscissa axis, as before, we plot time. These graphs show the variation of velocity with time, i.e. the time dependence of velocity. For uniform rectilinear motion, this "dependence" consists in that velocity remains unchanged in the course of time. Therefore, the corresponding velocity graph is the straight line parallel to the time axis (Fig. 29). Graph 1 in this figure describes the motion of the body in the positive direction of the  $X$ -axis. Graph 2 refers to the case when a body moves in the opposite direction (its velocity projection is negative).

The velocity graph can also be used for calculating the displacement of the body for a given interval of time. It is numerically equal to the area of the hatched rectangle (Fig. 30). Indeed, the area of a rectangle is equal to the product of its two adjacent sides. But one side of this rectangle is time  $t$  on

<sup>11</sup> The graph of the velocity projection is called for brevity the velocity graph.

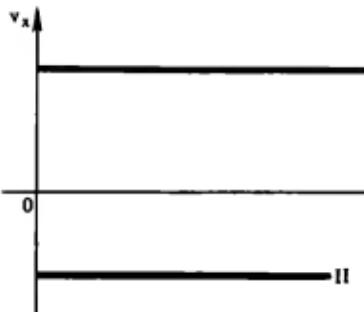


Fig. 29



Fig. 30

a certain scale, while the other is the projection of the velocity  $\vec{v}$ . Their product  $v_x t$  is just the projection of the displacement of the body.

?

1. Which motion is described by the graph shown by the dashed line in Fig. 28?
2. Which types of motion correspond to graphs I and 2 in Fig. 29?
3. How can we find the displacement of a body with the help of the graph of velocity projection?

#### EXAMPLE OF SOLVING A PROBLEM

Figure 31 represents the graphs of motion of a car and a bicycle. Using these graphs, find the place and time of their meeting.

*Solution.* Analyzing graph I, we see that the car moves uniformly along the

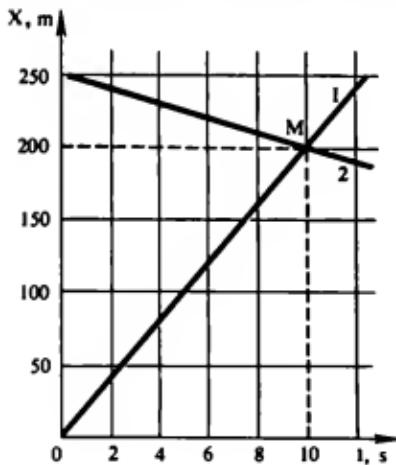


Fig. 31

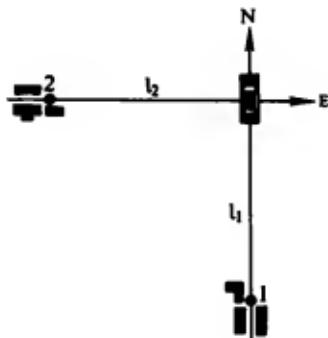


Fig. 32

*X*-axis at a speed of 20 m/s, while from graph 2 it follows that the cyclist moves towards the car uniformly at a speed of 5 m/s. It can also be seen from the figure that at the initial instant of time the car and the bicycle were at a distance of 250 m from each other. The graphs intersect at point *M*. This means that the car and the bicycle have met at this point. Their meeting occurred after 10 s from the time reference point at a distance of 200 m from the initial position of the car.

---

### Exercise 3

1. Using graphs 2 and 4 (see Fig. 28), find the distance between the moving bodies at the moment of time  $t = 3$  s.
2. Determine the magnitude and direction of the velocity of a body with the help of the graph shown in Fig. 27.

---

### Hometask

Using graphs 1 and 2 of the projections of velocities (see Fig. 29), plot the graph of their magnitudes.

---

## 1.8. Relative Nature of Motion

It was mentioned in Sec. 1.2 that the position of a body (point) in space is specified relative to some other object chosen as the reference body. For this purpose, coordinate axes are drawn through the reference body. It is said that the coordinate system is fixed to this body.

However, we may take for the reference body any object and attach to it its own coordinate system. Then the position of the same body can be simultaneously considered in different reference systems. Clearly, the same body may have quite different coordinates relative to different reference frames in different coordinate systems. For example, the position of a car on the road can be defined by indicating that it is at a distance  $l_1$  to the North of locality 1 (Fig. 32). On the other hand, we can say that the car is at a distance  $l_2$  to the East of locality 2. Consequently, the *position of a body is a relative quantity: it is different for different reference bodies and coordinate systems fixed to them*.

But not only the position of a body is a relative quantity. *Its motion is also relative. What is the essence of relativity of motion?*

A child who watched for the first time floating of ice from the bank of a river asked a question: "What are we riding on?" Obviously, he "chose" an ice-floe as the reference body. Being at rest relative to the reference frame fixed to the bank, the boy moved with it relative to the reference system – the ice-floe – "chosen" by him.

Let us consider another well-known example of the relativity of motion. Everybody knows how it is difficult to establish which of the trains is moving and which is at rest when you are in the train and watch from the

window another train passing by. Strictly speaking, if you see only the other train without seeing the ground, buildings, clouds, etc., it is impossible to say which of the trains is moving uniformly and rectilinearly. If the passenger of one train states that "his" train is in motion and the other is at rest, the passenger of the second train can rightfully say that it is "his" train that is in motion, while the other is at rest. Essentially, both passengers are right since motion is relative.

In actual practice, we must often consider the motion of the same body relative to different reference bodies which *are moving themselves relative to one another*. For example, a gunner must know not only the motion of the shell relative to the Earth relative to which the gun is at rest. He must also know how the shell moves relative to the tank at which it is fired and which moves relative to the Earth. A pilot should have information concerning the flight of the aeroplane relative to the Earth as well as relative to the air which is also in motion, etc.

**MOTION FROM DIFFERENT POINTS OF VIEW.** Let us consider the motion of the same body in two different reference systems which move relative to each other. We assume that one of these systems is fixed while the other moves uniformly in a straight line relative to this system. We can illustrate this by a simple example. A boat crosses a river at right angles to the flow relative to which it moves at a certain velocity.

Suppose that the motion of the boat is being watched by two observers. One of them is at point  $O$  on the bank (Fig. 33), while the other is floating with the current on a raft. Both the observers measure the displacement of the boat and the time over which this displacement occurs.

The raft is at rest relative to water, but moves relative to the bank with the velocity of flow. We draw an imaginary system of coordinates  $XOY$  through point  $O$ . The  $X$ -axis is directed along the bank, and the  $Y$ -axis is perpendicular to the flow (see Fig. 33). This is a *fixed* reference system. We also associate a system of coordinates  $X'O'Y'$  with the raft, the  $X'$ - and  $Y'$ -axes being parallel to the  $X$ - and  $Y$ -axes respectively. This is a *movable* reference system.

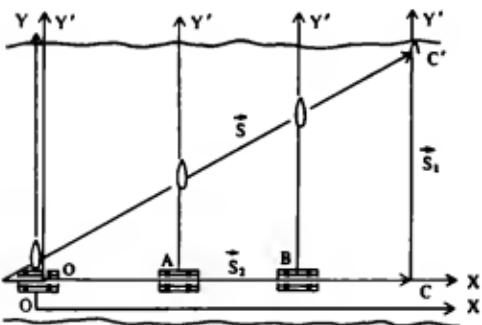


Fig. 33

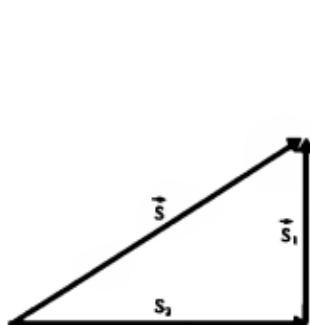


Fig. 34

Let us now consider the motion of the boat relative to the two observers, i.e. relative to the two reference systems.

The observer on the raft, who is moving together with "his" reference system downstream, finds that the boat is moving away from him perpendicularly to the flow all the time. He observes the same pattern at point  $A$ , point  $B$  (Fig. 33) or at any other point. When the raft reaches a certain point  $C$  after a time  $t$ , the boat touches the opposite bank at point  $C'$ . Relative to the moving reference system (raft), the boat is displaced by  $\bar{s}_1 = CC'$ . Dividing this displacement by the time  $t$ , the moving observer will obtain the following expression for the velocity  $\bar{v}_1$  of the boat relative to the raft (the river flow):

$$\bar{v}_1 = \bar{s}_1/t.$$

To the fixed observer on the bank, the motion of the boat appears to be quite different. Relative to "his" reference system, the boat is displaced by  $\bar{s} = OC$  over the same time  $t$ . During this interval of time, the moving system attached to the raft has been displaced by  $\bar{s}_2$  (the boat is said to be carried away downstream).

The displacement of the boat is schematically shown in Fig. 34.

**SUMMATION OF DISPLACEMENTS.** It can be seen from Figs. 33 and 34 that the displacement  $\bar{s}$  relative to the fixed reference system is connected to the displacements  $\bar{s}_1$  and  $\bar{s}_2$  through the formula

$$\bar{s} = \bar{s}_1 + \bar{s}_2. \quad (1.8.1)$$

**SUMMATION OF VELOCITIES.** Dividing the displacement  $\bar{s}$  by the time  $t$ , we obtain the velocity  $\bar{v}$  of the boat relative to the fixed reference system:

$$\bar{v} = \frac{\bar{s}}{t} = \frac{\bar{s}_1 + \bar{s}_2}{t} = \frac{\bar{s}_1}{t} + \frac{\bar{s}_2}{t},$$

or

$$\bar{v} = \bar{v}_1 + \bar{v}_2, \quad (1.8.2)$$

where  $\bar{v}_2 = \bar{s}_2/t$  is the velocity of the raft relative to the bank (i.e. the flow velocity). This formula represents the velocity summation rule: the velocity of a body relative to a fixed reference system is equal to the geometrical sum of its velocity relative to a moving reference system and the velocity of the moving system relative to the fixed one.

Thus, a body has different displacements and velocities in different reference systems. Its trajectories are also different ( $CC'$  relative to the moving system and  $OC$  relative to the fixed system). This is the essence of the relative nature of motion.

**THE STATE OF REST IS ALSO RELATIVE.** The example of a moving and a fixed train shows that not only motion but also the state of rest is relative. If a body is at rest relative to some reference system, we can always

find reference systems relative to which the body is in motion. There are no bodies in a state of "absolute rest". Motion is inherent in all bodies, viz. everything that exists in nature, in the whole material world.

**A CORRECTION WHICH IS IMPORTANT IN PRINCIPLE BUT NOT ALWAYS NECESSARY.** In the above example with the boat and raft, both observers on the bank and on the raft measure the interval of time during which the motorboat crosses the river, from "start" to "finish". We assume that the time interval measured by the two observers is the same. In other words, it is assumed that the time interval between the events "start" and "finish" does not change as we go over from the fixed reference system to the moving one. As a matter of fact, this is not true. According to Einstein's theory of relativity, which is now believed to describe motion more accurately, the time between two events, measured by an observer in motion relative to the place where the events occur, is longer than the time between the same events measured by an observer who is at rest relative to this place. This means that motion retards the passage of time. Consequently, the formula for summation of velocities acquires the form differing from (1.8.2). However, time dilation becomes noticeable only at velocities close to  $3 \times 10^8$  m/s (the velocity of light). The velocities with which we have to deal in everyday life are always negligibly small in comparison with this velocity. For such motions, formula (1.8.2) is found to be quite satisfactory.

?

1. What is the essence of the relativity of motion?
2. How do water and the bank move relative to the boat in the example considered above?
3. A combine harvester moves in a field at a speed of 2.5 km/hr relative to the Earth and loads a track with grain without coming to a halt. Relative to which reference bodies is the track in motion and at rest?
4. A tugboat tows a barge on a river. Relative to which reference bodies is the barge in motion? Relative to which body is it at rest?

#### EXAMPLES OF SOLVING PROBLEMS

1. A swimmer whose velocity  $v_1$  relative to water is 5.00 km/hr crosses the river of width  $l = 120$  m, swimming at right angles to the flow. The flow velocity  $v_2 = 3.24$  km/hr. How much time does it take the swimmer to cross the river? What is his displacement and velocity relative to the bank?

*Solution.* In the coordinate system attached to water, the swimmer moves all the time at right angles to the direction of the flow with the velocity  $v_1$ . The magnitude of his displacement  $s_1$  is equal to the river width:  $s_1 = l$ . The time  $t$  the swimmer moved can be found from the relation  $l = v_1 t$ :

$$t = \frac{l}{v_1}$$

(this formula shows that the time  $t$  does not depend on the flow velocity  $v_2$ ).

The motion of the swimmer relative to the bank is different. The displacement  $s$  of the swimmer relative to the bank is the sum of its

displacement  $\bar{s}_1$  relative to water and the displacement  $\bar{s}_2$  of water itself relative to the bank:

$$\bar{s} = \bar{s}_1 + \bar{s}_2.$$

The magnitude of the displacement  $\bar{s}_2$  can be found from the equality  $s_2 = v_2 t$ . Substituting for  $t$  its value  $l/v_1$ , we get

$$s_2 = \frac{v_2}{v_1} l.$$

From the displacement vector triangle (Fig. 35), we have

$$s = \sqrt{s_1^2 + s_2^2}.$$

Since  $s_1 = l$  and  $s_2 = (v_2/v_1)l$  we obtain

$$s = \sqrt{l^2 + \left(\frac{v_2}{v_1} l\right)^2} = l \sqrt{1 + \left(\frac{v_2}{v_1}\right)^2}.$$

Substituting the values of  $l$ ,  $v_1$  and  $v_2$  given in the condition of the problem, we obtain  $s = 120 \text{ m} \sqrt{1 + (0.9/1.38)^2} \approx 143 \text{ m}$ .

The swimmer velocity relative to the bank can be found from the velocity triangle (see Fig. 35):

$$v = \sqrt{v_1^2 + v_2^2}, \quad v = \sqrt{(0.9 \text{ m/s})^2 + (1.38 \text{ m/s})^2} \approx 1.65 \text{ m/s}.$$

The same result is obtained from the equality

$$v = \frac{s}{t} = \frac{sv}{l}; \quad v = \frac{143 \text{ m} \cdot 1.38 \text{ m/s}}{120 \text{ m}} = 1.65 \text{ m/s}.$$

2. A swimmer (see Problem 1) wants to cross the river via the shortest path (from  $A$  to  $B$ , Fig. 36). How much time will he need for this?

*Solution.* The magnitude of the displacement  $\bar{s}$  of the swimmer relative to the bank is equal to the river width  $l$ ,  $s = l$ .

According to formula (1.8.1),  $\bar{s} = \bar{s}_1 + \bar{s}_2$ , where  $\bar{s}_1$  is the swimmer displacement relative to water and  $\bar{s}_2$  is the displacement of water relative to the bank. The corresponding vector triangle is shown in Fig. 36. On the same figure, we have the velocity triangle. We see from this triangle that the magnitude  $\bar{v}$  of the velocity relative to the bank can be found from the

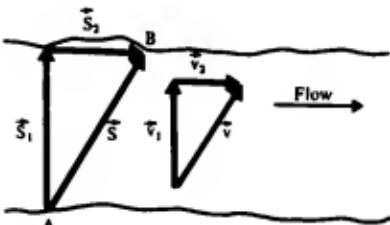


Fig. 35

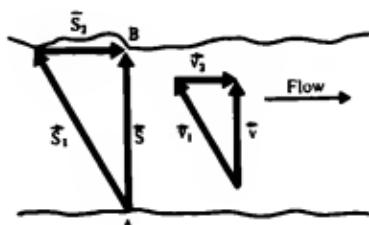


Fig. 36

equality

$$v = \sqrt{v_1^2 - v_2^2}; \quad v = \sqrt{(1.38 \text{ m/s})^2 - (0.90 \text{ m/s})^2} = 1.04 \text{ m/s.}$$

Hence we obtain the following expression for  $t$ :

$$t = \frac{l}{v}; \quad t = \frac{120 \text{ m}}{1.04 \text{ m/s}} = 115 \text{ s.}$$

It follows from the expression for velocity  $v$  that if the velocity of the swimmer relative to water is less than the flow velocity, the swimmer cannot cross the river via the shortest path (the radicand is negative in this case!).

---

#### Exercise 4.

1. The engine of an aeroplane imparts to it a velocity of 900 km/hr relative to air. What is its velocity relative to the Earth for fair wind (head wind) whose velocity is 50 km/hr?
2. A motorcar moves westwards at a velocity of 80 km/hr. Another car moves at the same velocity towards the first car. At a certain moment, the distance between the cars is 10 km. What time does it take them to meet?
3. An aeroplane starting from Moscow keeps a course northwards at a height of 8 km and velocity 720 km/hr. What will be its coordinates relative to the airport in 2 hr after the beginning of the flight if the westerly wind is blowing at a speed of 10 m/s?
4. Does the current prevent the swimmer from crossing the river? Does the current prevent the swimmer from crossing the river along the shortest path?

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## 1.9. On System of Units

It follows from the above discussion of motion that its analysis requires the knowledge of at least two quantities: displacement and time.

The magnitude of displacements, as well as periods of time, are expressed by numbers. These numbers are obtained as a result of measurements.

*To measure a quantity means to compare it in some way with a similar quantity conditionally taken as a unit*

We can, for example, measure the length of the corridor in school by comparing it with the length of a step. Counting the number of steps over the length of the corridor, we find how many times the length of the corridor is larger than the length of a step. This number (of times) expresses the length of the corridor in steps.

Consequently, we must first of all choose a unit for the quantity being measured. This can be done quite arbitrarily. For instance, quite different

units were used for measuring length in different times and in different countries. The units of length were the length of step, the length of foot, the distance from the elbow to the tip of the middle finger, the distance covered by a traveller per day, etc. Reading the lines in the comedy "Gore ot Uma" (Misfortune from Wisdom)

I would keep these gentlemen  
Within a gunshot of the capital!

we are aware that a character of this comedy, Famusov, uses for a unit of length the distance covered by a shell fired from a cannon. This peculiar measure of length was often used in old times by military men.

At present, the universal unit of length is adopted all over the world. This is a metre (m).

*One metre is the distance between two cuts on a specially shaped bar made of a platinum-iridium alloy.*

This rod is said to be a *standard* of length. It is stored in the International Bureau of Weights and Measures in France. The copies of the standard metre are kept in all countries. They are used for graduating numerous rulers intended for measuring length.

Besides the basic unit of length, viz. the metre, the units which are 10, 100, 1000, etc. times longer or shorter than a metre are widely used (e.g. kilometre, centimetre, millimetre, micrometre).

The unit of time can also be chosen arbitrarily. Naturally, it is impossible to make a standard of time in the form of a certain body like a ruler-meter. *A standard of time must be the duration of a certain regularly repeating process.* At present, the motion of the Earth around the Sun is taken for such a process: one revolution is performed by the Earth during one year. However, the unit of time is not a year but a definite part of this time interval, viz. a second. One year contains  $31\ 556\ 925.9747$  seconds (for very rough calculations, it can be assumed that 1 year =  $\pi \times 10^7$  s).

In everyday life and in engineering, other units of time are used: minute (min) and hour (hr) (1 min = 60 s and 1 hr = 3600 s).

Besides length and time, we have introduced one more quantity, the velocity of motion. Do we need a special unit for this quantity?

It turns out that it is not necessary since velocity, as we already know, is connected with length and time through the following relation:

$$v = \frac{s}{t}.$$

This formula shows that if a body performs a one-metre displacement during one second, the velocity of the body is equal to unity (1 m/s). The velocity of such motion can be taken for a unit of velocity.

*The unit of velocity is the velocity of such a uniform rectilinear motion in which the displacement of a body during one second is one metre*

There is also no need in choosing a special unit for measuring, for example, volume, since volume is connected with length and can be measured in cubic metres. Then when do we need a special unit of measurement and when do we not?

There exist certain relations between physical quantities because all natural phenomena are connected in a certain way with one another. Relations between quantities are expressed in the form of mathematical formulas. The same formulas relate units of measurement of physical quantities. Therefore, the units can be expressed in terms of the units of other quantities.

We can choose a small number of (basic) quantities and establish the units of their measurement in an arbitrary way. The units for other (*derived*) quantities can then be established on the basis of the mathematical formulas that connect the derived quantities with the basic ones.

The set of thus established units for all physical quantities is called the *system of units*.

There are several different systems of units. This division is determined by the physical quantities taken as basic and by the choice of the units for them.

At present, the *International System of Units* (SI) is adopted. It is constructed on the basis of seven quantities, including length and time. The SI unit of length is one metre and the unit of time is a second. The remaining basic quantities of SI and their units of measurement will be introduced later.

Obviously, the definition of the unit of velocity (1 m/s) given above pertains to SI.

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## Summary

The phenomenon of mechanical motion of bodies (material points) consists in that the position of a body relative to other bodies (i. e. its coordinates) changes with time.

To find the coordinates of a body at any instant of time, we must know the initial coordinates and the displacement vector. The change in the coordinate is equal to the projection of the displacement vector onto the corresponding coordinate axis.

The simplest type of motion is uniform rectilinear motion. For this motion, only one coordinate has to be determined since the coordinate axis can be directed along the displacement. The  $x$ -coordinate of the body at any instant of time  $t$  can be calculated from the formula

$$x = x_0 + v_x t,$$

where  $x_0$  is the initial coordinate of the body and  $v_x$  is the projection of the velocity vector onto the  $X$ -axis. When this formula is used for calculations, the signs of the quantities appearing in it are determined by the conditions of the problem.

Mechanical motion is of relative nature. This means that the displacement and velocity of a body in different coordinate systems moving relative to each other are different. The trajectories and path lengths are also different.

The state of rest is also relative. If a body is at rest relative to some reference system, there exist other reference systems relative to which it moves.

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**VELOCITY MAY CHANGE**

Uniform rectilinear motion in which the displacement linearly depends on time according to the formula  $\bar{s} = \bar{v}t$  is not frequently encountered in actual practice. More often we have to deal with a motion in which displacements are different over equal intervals of time, and hence the velocity varies with time. Such a motion is called *nonuniform*.

*The motion in which a body covers unequal distances during equal intervals of time is called nonuniform motion.*

Trains, motorcars, aeroplanes, etc. generally move nonuniformly. For nonuniform motion, the formula  $\bar{s} = \bar{v}t$  cannot be used for determining the displacement, since the velocity is different at different points of the trajectory. Consequently, we should be able to calculate the velocity at any point and at any instant of time, otherwise we cannot calculate the displacement and hence the coordinate of the body. How are displacements and velocities calculated for a nonuniform motion? What must we know for that?

**2.1****Velocity of Nonuniform Motion**

**AVERAGE VELOCITY.** This concept is sometimes used while considering a nonuniform motion.

If a body was displaced by  $\bar{s}$  during a time interval  $t$ , then the average velocity is obtained by dividing  $\bar{s}$  into  $t$ :

$$\bar{v}_{av} = \frac{\bar{s}}{t}.$$

Thus, the *average velocity is the average displacement of the body per unit time.*<sup>11</sup>

If, for example, a train moving in a straight line covers 600 km during 10 hr, this means that on the average, it covers 60 km every hour. Obviously, during some time the train did not move at all but had a stop. It increased

<sup>11</sup> While considering the average velocity, for example, of a car or a pedestrian, the scalar quantity determined by the length of the path the body covers on the average per unit time,  $v_{av} = l/t$ , is often meant rather than the vector  $\bar{v}_{av} = \bar{s}/t$ .

its speed while leaving the station and decreased the speed while approaching it. All this is disregarded when we determine the average velocity, assuming that the train covers 60 km per hour, or 30 km in 30 min, and so on. Using the formula  $\bar{v}_{av} = \bar{s}/t$ , we assume that the train moves uniformly at a constant velocity equal to  $\bar{v}_{av}$ , although it may happen there were not a single hour when the train covered exactly 60 km.

If we know the average velocity, the displacement can be determined from the formula

$$\bar{s} = \bar{v}_{av} t.$$

It should be remembered that this formula gives the correct result only for the part of the path on which the average velocity has been determined. If we use the value 60 km/hr for the average velocity to calculate the displacement of the train not over 10 hr but over 2, 4 or 5 hr, we shall obtain a wrong result. This is explained by the fact that the average velocity over the time of 10 hr is not equal to the average velocities over 2, 4 or 5 hr.

Consequently, the average velocity generally does not allow us to calculate the displacement and coordinates of a moving body at any instant of time. Nevertheless, the concept of velocity can be used for nonuniform motion as well since mechanical motion is a *continuous process*.

**INSTANTANEOUS VELOCITY.** The continuity of motion consists in the following. If, for instance, a body (or point), moving rectilinearly with increasing velocity, has passed from point *A* to point *B*, it must pass through all intermediate points between *A* and *B*, without any gaps. Moreover, let us suppose that the body arrives at point *A* moving uniformly at a velocity of 5 m/s, and after passing point *B* it also moves uniformly but at a velocity of 30 m/s. Segment *AB* was covered during 15 s. Consequently, the velocity has changed over segment *AB* by 25 m/s during 15 s. But as the body could not miss any point on its route, its velocity must have acquired all the values between 5 and 30 m/s, and without skipping any intermediate value! This is the essence of the continuity of mechanical motion: *neither the coordinates of the body nor its velocity may change abruptly*. Hence it follows a very important conclusion. There is an infinite set of velocity values in the interval from 5 to 30 m/s (infinitely large number, as it is said in mathematics). But between points *A* and *B* there is an infinite set (infinitely large number) of points, while the 15-second time interval during which the body has moved from *A* to *B* consists of an infinitely large number of instants of time (time also passes smoothly!).

Consequently, the velocity of the body has a definite value at each point of the trajectory and at each instant of time.

*The velocity of a body at a given instant of time or at a given point is called the instantaneous velocity*

The velocity of a uniform rectilinear motion is equal to the ratio of its displacement to the time interval over which this displacement took place. This ratio also defines the average velocity of nonuniform motion. The same ratio will help us to understand the meaning of instantaneous velocity as well.

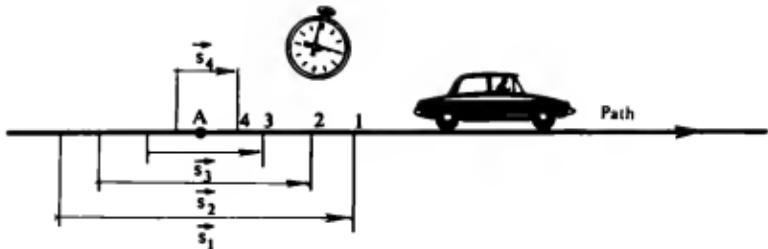


Fig. 37

Let us suppose that a body (as usual, we mean a certain point of this body) is moving rectilinearly but not uniformly. How can we calculate the instantaneous velocity of the body at a certain point  $A$  of its trajectory? We isolate a small segment  $1$  on this trajectory, which contains the point  $A$  (Fig. 37). We denote by  $s_1$  a small displacement of the body on this segment and the small time interval over which it occurs by  $t_1$ . Dividing  $s_1$  by  $t_1$ , we obtain the average velocity over this segment: the velocity varies continuously and hence it is different at different points of segment  $1$ .

Let us now reduce the length of segment  $1$ . We choose a segment  $2$  which also includes point  $A$ . The displacement over this segment is equal to  $s_2$  ( $s_2 < s_1$ ) and corresponds to the time interval  $t_2$ . Obviously, the velocity of the body can change by a smaller value over segment  $2$ . The ratio  $s_2/t_2$  gives us the average velocity for this smaller segment. If we take a segment  $3$  which also includes point  $A$  and is smaller than segments  $1$  and  $2$ , the change in the velocity over this segment will be still smaller. Dividing the displacement  $s_3$  by the time interval  $t_3$ , we again obtain the average velocity on this small part of the trajectory.

Let us gradually reduce the time interval over which the displacement of the body is being considered. The displacement of the body will simultaneously decrease. Ultimately, the segment of the path adjoining point  $A$  contracts to the very point  $A$ . It is at this stage that the average velocity becomes equal to the instantaneous velocity at point  $A$  where the body is located at a given moment. This is so because the change in the velocity over a very small segment will be so small that it need not be taken into account. In other words, we can assume that the velocity does not change here (the motion can be treated as uniform).

*The instantaneous velocity, or the velocity at a given point is equal to the ratio of a sufficiently small displacement over a part of path containing this point to the small time interval during which this displacement occurred*

*The instantaneous velocity is a vector quantity.* Its direction coincides with the direction of displacement (motion) at a given point. Henceforth, while speaking about the velocity of nonuniform motion, we shall mean the instantaneous velocity.

Thus, the technique used for explaining the meaning of instantaneous velocity consists in the following. We imagine a gradual reduction of the path segment and the time during which it is traversed, until we can

distinguish neither the segment from a point nor the nonuniform motion from uniform. This technique is always used for investigating phenomena in which some *continuously* varying quantities take part.<sup>2)</sup>

It only remains for us to find out which quantities are required for determining the instantaneous velocity of a body at any point of its trajectory at any instant of time.

---

?

1. What is an average velocity?
2. Is it possible to find the displacement of a body over any part of a time interval for which we know the average velocity of motion?
3. What is the essence of continuity of motion?
4. What is meant by an instantaneous velocity?

---

### Exercise 5

1. While travelling from one location to another, a motorcar had a constant velocity of 60 km/hr over half the time. What is its constant velocity during the remaining time of travel if its average velocity is 65 km/hr?
2. A motorcar covers the first half of the distance to its destination at a constant velocity of 50 km/hr, and the second half at a constant velocity of 60 km/hr. Find the average velocity of the car.

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## 2.2.

### Acceleration. Uniformly Accelerated Motion

The instantaneous velocity of a body changes continuously during nonuniform motion from point to point and from one instant of time to another. How can we then calculate the instantaneous velocity of the body?

It was shown above that the coordinate of a body can be calculated at any instant of time if we know the rate of its time variation. In the same way, the velocity can be calculated at any instant of time if we know the rate of its variation, i.e. change in velocity per unit time.

**UNIFORMLY ACCELERATED MOTION.** For the sake of simplicity, we consider a nonuniform motion of a body in which its velocity changes by the same value over any equal intervals of time. Such a motion is said to be uniformly accelerated.

*The motion of a body during which its velocity changes by the same value over any equal time intervals is called a uniformly accelerated motion.*

If  $\bar{v}_0$  is the velocity of a body at a certain initial instant of time, and  $\bar{v}$  is its velocity after a time  $t$ , the change in velocity per unit time is  $(\bar{v} - \bar{v}_0)/t$ .

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<sup>2)</sup> A brief but vivid description of this method, which forms the basis of Differential Calculus, can be found on the first page of Part 3, Vol. 3 of the novel "War and Peace" by Leo Tolstoy.

This quantity characterizes the rate of variation of velocity and is called *acceleration*.

Since acceleration is the product of the vector quantity  $\bar{v} - \bar{v}_0$  by the scalar  $1/t$ , it is a vector quantity (see Sec. 1.4). Acceleration is denoted by  $\bar{a}$ :

$$\bar{a} = \frac{\bar{v} - \bar{v}_0}{t}. \quad (2.2.1)$$

*The acceleration of a uniformly accelerated body is a constant quantity equal to the ratio of the change in its velocity to the interval of time during which this change occurred.*

If the magnitude of acceleration is large, the body rapidly gains velocity during acceleration (or rapidly loses velocity during retardation).<sup>11</sup>

The velocity  $\bar{v}$  of a body at any instant of time can be found if we know its initial velocity  $\bar{v}_0$  and acceleration  $\bar{a}$ . Indeed, it follows from formula (2.2.1) that

$$\bar{v} = \bar{v}_0 + \bar{a}t. \quad (2.2.2)$$

*To determine the instantaneous velocity  $\bar{v}$  of a body, we need to know its acceleration.*

In what units is acceleration measured?

Since  $\bar{a} = (\bar{v} - \bar{v}_0)/t$ , the magnitude of acceleration is equal to unity if the magnitude of the change in velocity is equal to unity over a unit interval of time. Hence, the *SI unit of acceleration is the acceleration of a uniformly accelerated body whose velocity changes by 1 m/s during 1 s*. Consequently, acceleration in SI is expressed in metres per second per second or in metres per second squared ( $\text{m/s}^2$ ).

**PROJECTIONS OF VELOCITY AND ACCELERATION.** It was mentioned above that we can use in calculations the equations containing the projections of vectors onto coordinate axes and not the vectors themselves.

In a rectilinear motion, vectors  $\bar{v}_0$  and  $\bar{v}$  are directed along the same straight line coinciding with the trajectory of motion. It is convenient to direct a coordinate axis (e.g. the  $X$ -axis) along this straight line.

We proved in Sec. 1.4 that the projection of the sum of two vectors onto an axis is equal to the sum of their projections onto the same axis. We denote the projections of vectors  $\bar{v}$ ,  $\bar{v}_0$  and  $\bar{a}$  onto the  $X$ -axis by  $v_x$ ,  $v_{0x}$  and  $a_x$ . Then from Eq. (2.2.2) we obtain

$$v_x = v_{0x} + a_x t. \quad (2.2.3)$$

Since the three vectors  $\bar{v}$ ,  $\bar{v}_0$  and  $\bar{a}$  lie on the same straight line (the  $X$ -axis), the magnitudes of their projections are equal to the magnitudes of

<sup>11</sup> If the velocity of the body changes nonuniformly over equal intervals of time, instantaneous acceleration should be used. The value of instantaneous acceleration is found by using the same method that was adopted for determining instantaneous velocity.

the vectors themselves, while their signs are determined by the direction of these vectors relative to the chosen axis.

If the signs of the projections of vectors  $\vec{v}_0$  and  $\vec{a}$  coincide, the magnitude of the velocity of the body increases with time, i.e. the body is accelerated. If, however, the signs of the projections of  $\vec{v}_0$  and  $\vec{a}$  are opposite, the magnitude of the velocity of the body decreases with time, which means that the body is decelerated.

When a body moves with increasing velocity, vectors  $\vec{v}$ ,  $\vec{v}_0$  and  $\vec{a}$  have the same direction; for decelerated motion, the vector  $\vec{a}$  is opposite to vectors  $\vec{v}$  and  $\vec{v}_0$ .

**MOTION DURING DECELERATION.** If the velocity of a body decreases with time (the body is decelerated), at a certain instant the velocity may become equal to zero. How will the body move after that? Obviously, if some varying quantity passes through the zero value, it changes its sign. In our case, the varying quantity that changes its sign is the velocity. This means that after the velocity of a body has acquired the zero value, the body starts to move in the opposite direction (see Problem 2 on p. 45).

The motion at a velocity that increases in magnitude is usually called the accelerated motion, and the motion with a decreasing velocity is decelerated. In mechanics, however, any rectilinear nonuniform motion is referred to as accelerated. Irrespective of whether a motorcar starts moving or is braking, it moves with an acceleration. Rectilinear accelerated motion differs from decelerated motion only in the sign of the projection of the acceleration vector onto a chosen axis.

?

1. What is acceleration and what is it required for?
2. The velocity changes when a body is in a nonuniform motion. How is this change characterized in terms of acceleration?
3. What is the difference between "decelerated" and "accelerated" rectilinear motion?
4. Give the definition of a uniformly accelerated motion.
5. Can a body move with a high velocity but with a small acceleration?
6. How is the acceleration vector directed when a body is in nonuniform rectilinear motion?
7. Velocity is a vector quantity, both its magnitude and direction may change. Which of them is changing when a body is in uniformly accelerated rectilinear motion?
8. Can the velocity of a body be equal to zero at the moment when its acceleration is other than zero?

## EXAMPLES OF SOLVING PROBLEMS

1. A motorcar passes by an observer when its velocity is 10 m/s. At this moment, the driver applies the brakes and the car starts moving with an acceleration of  $1.0 \text{ m/s}^2$ . What time will it take the car to stop?

*Solution.* We choose for the coordinate origin the point where the observer is located and direct the  $X$ -axis along the trajectory of the car (Fig. 38). We

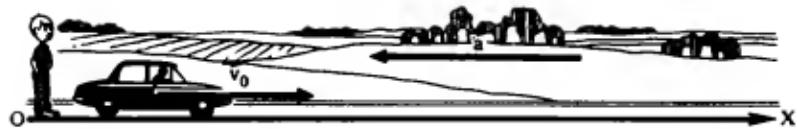


Fig. 38

denote by  $\bar{v}_0$  the velocity of the car at the moment it passes by the observer and by  $\bar{a}$  the acceleration as a result of applying the brakes.

The time of motion of the car until it stops can be calculated from the formula

$$v_x = v_{0x} + a_x t,$$

where  $v_x$ ,  $v_{0x}$  and  $a_x$  are respectively the projections of the final and initial velocity of the car and its acceleration onto the  $X$ -axis.

The car moves along the  $X$ -axis, and hence  $v_{0x} = v_0$ . Since its velocity decreases,  $a_x = -a$ . At the initial moment,  $v_x = 0$ . Consequently,

$$0 = v_0 - at \quad \text{or} \quad at = v_0.$$

This gives

$$t = \frac{v_0}{a}.$$

Substituting the values of  $v_0$  and  $a$  into this expression, we get

$$t = \frac{10 \text{ m/s}}{1.0 \text{ m/s}^2} = 10 \text{ s}.$$

The car will stop after 10 s from the beginning of braking.

2. A body moves rectilinearly with a gradually decreasing velocity. The acceleration  $\bar{a}$  is constant and equal in magnitude to  $4 \text{ m/s}^2$ . At a certain instant of time, the magnitude of the velocity of the body  $v_0 = 20 \text{ m/s}$ . Find the velocity of the body in  $t_1 = 4 \text{ s}$  and  $t_2 = 8 \text{ s}$  after this moment.

*Solution.* We direct the  $X$ -axis along the initial velocity. Then the projection  $v_{0x}$  is positive and equal in magnitude to the vector  $\bar{v}_0$ :  $v_{0x} = v_0$ . Since the velocity is decreasing, the projection of the acceleration  $a_x$  is negative and equal to  $-a$ ,  $a_x = -a$ .

In order to find the projection of velocity,  $v_x$ , at required instants of time, we use the formula

$$v_x = v_{0x} + a_x t.$$

Hence for the moment of time  $t_1$  we have

$$v_{1x} = v_0 - at_1, \quad v_{1x} = 20 \text{ m/s} - 4 \text{ m/s}^2 \cdot 4 \text{ s} = 4 \text{ m/s},$$

and for  $t_2$

$$v_{2x} = v_0 - at_2, \quad v_{2x} = 20 \text{ m/s} - 4 \text{ m/s}^2 \cdot 8 \text{ s} = -12 \text{ m/s}.$$

The minus sign indicates that by the end of the eighth second the body moves in the direction opposite to the initial one.

Naturally, before the body starts moving in the opposite direction it should stop. We can easily find the moment  $t'$  at which this happened. At this instant, the projection  $v_x$  of the velocity is equal to zero, and hence  $v_{0x} = -a_x t'$ . This gives

$$t' = \frac{v_{0x}}{a_x}, \quad t' = -\frac{20 \text{ m/s}}{-4 \text{ m/s}^2} = 5 \text{ s.}$$

The direction of motion has reversed in 5 s from the instant when the velocity of the body was 20 m/s.

An example of such a motion is the motion of a body which has been pushed upwards along an inclined plane by imparting an initial velocity to it.

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### Exercise 6

1. A starting trolleybus moves at a constant acceleration of  $1.5 \text{ m/s}^2$ . In what time will it acquire the velocity of 54 km/hr?
2. A motorcar moving at a velocity of 36 km/hr stops due to braking during 4 s. Find its constant acceleration during braking.
3. A lorry moving with a constant acceleration has increased its velocity from 15 to 25 m/s. In what time did this take place if the acceleration of the lorry is  $1.6 \text{ m/s}^2$ ?
4. Which velocity will be acquired by a body moving during 0.5 hr with an acceleration of  $10 \text{ m/s}^2$ , if its initial velocity is equal to zero?

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## 2.3.

### Displacement in Uniformly Accelerated Motion

The most important thing for us is to know how to calculate the coordinates of a body. This is the basic problem of mechanics. For solving this problem, we must be able to determine the displacement of the body. How can we do this in the case of uniformly accelerated motion?

The formula for displacement can be easily obtained with the help of the graphic method.

It was shown in Sec. 1.7 that the displacement of a body in a uniform rectilinear motion is numerically equal to the area of the figure (rectangle) lying below the velocity graph. Is this valid for uniformly accelerated motion?

In uniformly accelerated motion of a body along the  $X$ -axis, the velocity does not remain constant but varies with time according to the formula

$$v_x = v_{0x} + a_x t.$$

Therefore, the graphs of the velocity projection has the form shown in Fig. 39. Line 1 in this figure corresponds to the motion with a positive

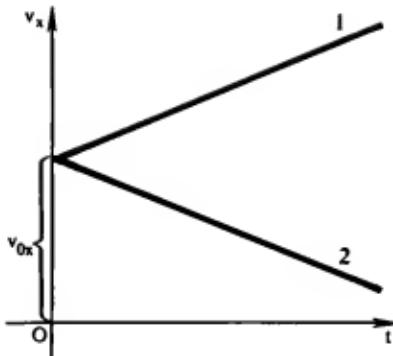


Fig. 39

projection of acceleration (the velocity is increasing), while line 2 describes the motion with a negative projection of acceleration (the velocity is decreasing). Both graphs refer to the case when the velocity of the body at the instant  $t = 0$  is  $\bar{v}_0$ .

**DISPLACEMENT CAN BE EXPRESSED IN TERMS OF AREA.** Let us isolate a small segment  $ab$  on the graph of velocity of a uniformly accelerated motion (Fig. 40) and drop from points  $a$  and  $b$  the perpendiculars onto the  $t$ -axis. The length of segment  $cd$  on the  $t$ -axis is numerically equal to the small interval of time over which the velocity has changed from its value at point  $a$  to that at point  $b$ . Under segment  $ab$  of the graph we have the narrow strip  $abcd$ .

If the interval of time numerically equal to segment  $cd$  is sufficiently small, the change in the velocity during this time is also small.

We can assume that the motion during such a small time interval is uniform, and the strip  $abcd$  differs from a rectangle only slightly. Therefore, the area of the strip is numerically equal to the projection of the displacement of the body during the time corresponding to segment  $cd$ .

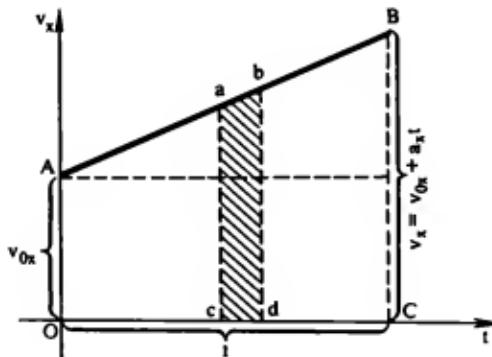


Fig. 40

We can split into such narrow strips the entire area of the figure lying below the velocity graph. Consequently, the displacement during the time  $t$  is equal in magnitude to the area of the trapezium  $OABC$ . As we know from geometry, the area of trapezium is equal to the product of the half-sum of its bases by the height. In the case under consideration, the lengths of the bases are equal to  $v_{0x}$  and  $v_x$ , while the height is equal to  $t$ . Hence, the projection  $s_x$  of the displacement

$$s_x = \frac{v_{0x} + v_x}{2} t. \quad (2.3.1)$$

We substitute into this formula the quantity  $v_{0x} + a_x t$  for  $v_x$ , which gives

$$s_x \frac{v_{0x} + v_{0x} + a_x t}{2} t = \frac{2v_{0x}t + a_x t^2}{2}.$$

Dividing the numerator by the denominator termwise, we get

$$\underline{\underline{s_x = v_{0x}t + \frac{a_x t^2}{2}}}, \quad (2.3.2)$$

while using this formula, it should be borne in mind that being the projections of vectors  $\vec{s}$ ,  $\vec{v}_0$  and  $\vec{a}$  onto the  $X$ -axis  $s_x$ ,  $v_{0x}$  and  $a_x$  may be positive or negative.

If the projection  $v_{0x}$  of the initial velocity is equal to zero, formula (2.3.2) becomes

$$s_x = \frac{a_x t^2}{2}.$$

The velocity graph for such a motion is shown in Fig. 41.

Now, when we have obtained the formula for displacement, we can easily derive the formula for calculating the  $x$ -coordinate of a body at any instant of time. It was shown above (see Sec. 1.5) that to find the  $x$ -coordinate at a certain instant of time  $t$ , we must add the projection of the displacement

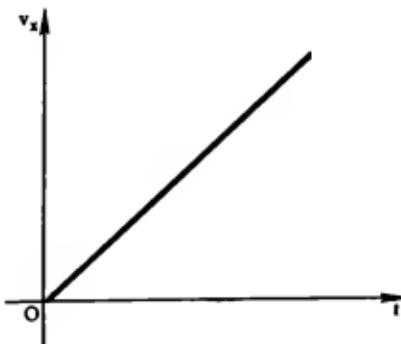


Fig. 41

vector onto the coordinate axis to the initial coordinate  $x_0$ :

$$x = x_0 + s_x.$$

Hence

$$\underline{x = x_0 + v_{0x}t + \frac{a_x t^2}{2}}. \quad (2.3.3.)$$

This formula is used for determining at any instant of time the position of a body in a uniformly accelerated rectilinear motion.

To find  $x$ , we must know the initial coordinate  $x_0$  and the velocity  $v_0$ , as well as the acceleration  $a$ .

Formulas (2.3.2) and (2.3.3) make it possible to describe a uniformly accelerated rectilinear motion in the same way as formula (1.6.3) allowed us to describe a uniform motion. It can be seen that for describing uniformly accelerated rectilinear motion one more quantity (acceleration) is required.

**ANOTHER FORMULA FOR DISPLACEMENT.** We can calculate the displacement of a uniformly accelerated body even when the time elapsed from the beginning of motion is unknown but we know the values of the initial and final velocities of the body. The formula for the displacement projection can be obtained from (2.3.1) and the formula  $v_x = v_{0x} + a_x t$ .

From this formula, we find the value of  $t$ :  $t = (v_x - v_{0x})/a_x$ , and substitute it into (2.3.1). This gives

$$s_x = \frac{v_x + v_{0x}}{2} \cdot \frac{v_x - v_{0x}}{a_x} = \frac{(v_x + v_{0x})(v_x - v_{0x})}{2a_x},$$

whence

$$\underline{s_x = \frac{v_x^2 - v_{0x}^2}{2a_x} \quad \text{and} \quad v_x^2 - v_{0x}^2 = 2a_x s_x}. \quad (2.3.4)$$

We have obtained the formulas for calculating displacement from the known values of the initial and final velocities and for the velocity at any point through which the moving body passes.

If the initial velocity of the body is equal to zero, we get

$$\underline{s_x = \frac{v_x^2}{2a_x} \quad \text{and} \quad v_x^2 = 2a_x s_x}. \quad (2.3.5)$$

9

1. What is the difference between the velocity graphs for uniform and uniformly accelerated motions?
2. How can we find the projection of the displacement in uniformly accelerated rectilinear motion of a body by using the graph of the velocity projection?

- What must we know to be able to calculate the coordinate of a body in uniformly accelerated rectilinear motion at any instant of time?
- Compare the time dependences of the magnitudes of displacement in uniform and uniformly accelerated motions of bodies. What is the difference between these dependences?

### EXAMPLES OF SOLVING PROBLEMS

1. The driver of a motorcar moving at a speed of 72 km/hr sees the red light and applies the brakes. As a result, the velocity of the car starts decreasing and it moves with a deceleration of  $5 \text{ m/s}^2$ . What is the distance covered by the car in the first two seconds after the brakes have been applied? Which distance will the car cover until it stops completely?

*Solution.* We direct the  $X$ -axis along the car trajectory (Fig. 42) and take for the coordinate origin the point on the road at which the brake was applied. For the time reference point, we take the instant when the driver has applied the brake.

The velocity  $v_0$  of the car is collinear with the  $X$ -axis, while its acceleration is opposite to it, so that the projection of the velocity  $v_0$  onto the  $X$ -axis is positive and the projection of the vector  $\vec{a}$  is negative. Hence,  $v_{0x} = v_0$  and  $a_x = -a$ .

The coordinate of the car can be found from the formula

$$x = x_0 + v_{0x} t + \frac{a_x t^2}{2}.$$

It follows from the condition of the problem that  $x_0 = 0$ ,  $v_0 = 20 \text{ m/s}$ ,  $a_x = -5 \text{ m/s}^2$ , and  $t = 2 \text{ s}$ . Consequently,

$$x = 0 + 20 \text{ m/s} \cdot 2 \text{ s} + \frac{-5 \text{ m/s}^2 \cdot 4 \text{ s}^2}{2} = 40 \text{ m} - 10 \text{ m} = 30 \text{ m}.$$

Let us now find the distance covered by the car before it stops. For this we must know the time  $t_1$  of motion to the stop. It can be determined from

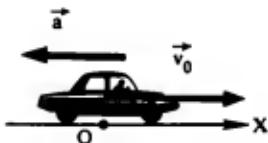


Fig. 42

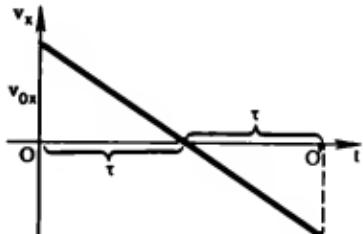


Fig. 43

the formula

$$v_x = v_{0x} + a_x t_1.$$

At this moment, the velocity is equal to zero, so that

$$0 = v_{0x} + a_x t_1 \text{ or } 0 = v_0 - a t_1,$$

whence

$$t_1 = \frac{v_0}{a}.$$

We substitute this expression for time into the formula for the coordinate

$$x = x_0 + v_{0x} t_1 + \frac{a_x t_1^2}{2}.$$

This gives

$$x = v_0 \frac{v_0}{a} - a \frac{(v_0/a)^2}{2} = \frac{v_0^2}{2a}.$$

Taking into account the values of the parameters given in the condition of the problem, we obtain

$$x = \frac{(20 \text{ m/s})^2}{2 \cdot 5 \text{ m/s}^2} = 40 \text{ m.}$$

2. Determine the displacement of a body whose velocity projection has the form shown in Fig. 43.

*Solution.* We calculate the projection  $s_x$  of the displacement by the formula

$$s_x = v_{0x} t + \frac{a_x t^2}{2}.$$

Figure 43 shows that for  $t = \tau$ , the velocity projection  $v_x$  is equal to zero. But  $v_x = v_{0x} + a_x t$ . Hence  $0 = v_{0x} + a_x \tau$ , which gives

$$a_x = - \frac{v_{0x}}{\tau}.$$

The total time of motion is  $2\tau$ . Consequently,

$$s_x = v_{0x} 2\tau + \frac{-v_{0x}(2\tau)^2}{2}$$

$$= 2v_{0x}\tau - \frac{4v_{0x}\tau}{2} = 0.$$

The answer shows that the graph represented in Fig. 43 corresponds to the same displacement of the body first in one direction and then in the direction opposite to it. As a result, the body turns out to be at the initial point.

3. When a train was approaching a station, the engine-driver switched off the engine of the locomotive, after which the train started to move with a constant acceleration of  $0.1 \text{ m/s}^2$ . Find the displacement of the train before

it stops if its velocity at the moment of switching off the engine was 20 m/s. In what time had the train stopped?

*Solution.* We direct the  $X$ -axis along the trajectory of the train. For the time reference point, we take the instant when the engine was switched off, while the coordinate origin will be the point where the train was at this moment.

The train velocity has the same direction as the  $X$ -axis, while its acceleration has the opposite direction, which means that the projection of the velocity  $v_0$  is positive and the projection of the acceleration  $\ddot{a}$  is negative. Consequently,  $v_{0x} = v_0$  and  $a_x = -a$ .

The projection  $s_x$  can be found by (2.3.4):

$$s_x = \frac{v_x^2 - v_{0x}^2}{2a_x}.$$

Substituting this expression the initial data and considering that  $v_x = 0$ , we get

$$s_x = \frac{0 - (20 \text{ m/s})^2}{-2 \cdot 0.1 \text{ m/s}^2} = \frac{400}{0.2} \text{ m} = 2000 \text{ m}.$$

The time of motion until the train stops is found from the formula

$$v_x = v_{0x} + a_x t.$$

Since  $v_x = 0$ , we have  $0 = v_0 - at$ , whence

$$t = \frac{v_0}{a}, \quad t = \frac{20 \text{ m/s}}{0.1 \text{ m/s}^2} = 200 \text{ s}.$$

---

### Exercise 7

1. Plot in the same coordinate axes the velocity graphs for two uniformly accelerated bodies one of which moves with the velocity increasing in magnitude and the other moves with a decreasing velocity. The initial velocities and accelerations of the bodies are 1 m/s, 0.5 m/s<sup>2</sup> and 9 m/s, 1.5 m/s<sup>2</sup> respectively. What is the distance covered by the second body until it stops? In what time will the velocities of the bodies become equal to each other? What is the distance covered by the first body during this time?
2. Figure 44 represents the graphs of the velocity projections for two moving bodies. What is the nature of motion of these bodies? What can you say about the velocities of these bodies at the moments of time corresponding to points *A* and *B*? Find the accelerations of the bodies and write the expressions for their velocities and displacements.
3. Using the graphs of the velocity projections for three

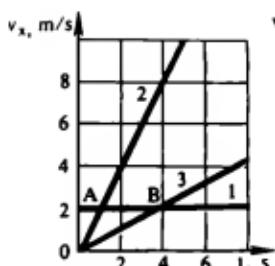


Fig. 44

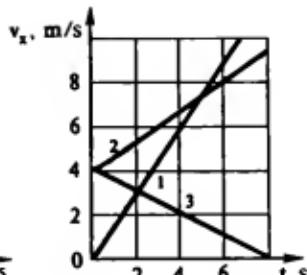


Fig. 45

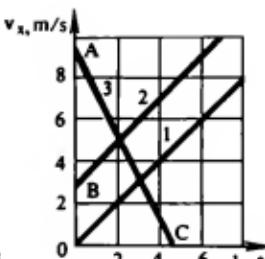


Fig. 46

bodies shown in Fig. 45, (a) determine the accelerations of these bodies; (b) for each body, write a formula describing the dependence of velocity on time and (c) find common and distinctive features of the motions corresponding to graphs 2 and 3.

- Figure 46 represents the graphs of the velocity projections of three moving bodies. Using these graphs, (a) find physical quantities corresponding to the intercepts  $OA$ ,  $OB$  and  $OC$  on the coordinate axes; (b) determine the accelerations of the bodies, and (c) write the expressions for velocity and displacement for each body.
- An aeroplane taxis along the runway for 15 s and takes off at a speed of 100 m/s. What is its acceleration and how long is the runway?
- A shell whose velocity is 1000 m/s pierces the wall of a dugout in  $10^{-3}$  s, as a result of which its velocity becomes 200 m/s. Find the thickness of the wall considering that the motion of the shell through the wall is uniformly accelerated.
- A rocket moves with an acceleration of  $45 \text{ m/s}^2$  and in a certain time attains the velocity of 900 m/s. Which distance will it cover during the next 2.5 s?
- At what distance from the Earth would a spaceship be in 30 minutes after its start if it moved all the time rectilinearly with an acceleration of  $9.8 \text{ m/s}^2$ ?
- It was observed that a race-horse attains its maximum velocity of 15 m/s after it has "gathered speed" for the first 30 m. What is a constant acceleration of the race-horse for this part of the distance?
- To be able to take off, an aeroplane has to gather a speed of 180 km/hr. At what distance from the starting point of the runway will the plane velocity attain this value if the acceleration is constant and equal to  $2.5 \text{ m/s}^2$ ?
- A train brakes and moves with an acceleration of

0.15 m/s<sup>2</sup>. At what distance from the spot where the brake was applied will the train velocity be equal to 3.87 m/s, if at the moment of braking the velocity was 54 km/hr?

#### Hometask

1. Compare formula (2.3.1) with the formula for displacement  $\bar{s} = \bar{v}_{avg}t$  (p. 40) and prove that the expression  $(v_{0x} + v_x)/2$  represents the projection of the average velocity in uniformly accelerated motion onto the  $X$ -axis
2. Using the graph of the velocity projection (see Fig. 43), plot the graph for the magnitude of the velocity

## 2.4. Measurement of Acceleration

One of the ways of the experimental determining of acceleration is the so-called *stroboscopic method*. It consists in illuminating a body which moves in the darkness in equal intervals of time by a flash of light. The corresponding device called *stroboscope* is shown in Fig. 47. Clearly, the body will be seen only in positions where it is illuminated. If the moving body is photographed (the shutter of the camera should be open during all the time of motion), consecutive positions of the body in equal intervals of time will be seen on the photographic film.

Figure 48 reproduces a stroboscopic photograph of the motion of a ball along an inclined plane for a time interval between flashes equal to 0.2 s. In order to find the acceleration with the help of this photograph, we must measure the lengths  $l_1$  and  $l_2$  of two neighbouring distances covered by the ball between the flashes. These lengths are equal to the magnitudes of displacements  $\bar{s}_1$  and  $\bar{s}_2$  over the time intervals  $\tau$  between flashes.

Writing the formulas for  $s_1$  and  $s_2$  and considering that the velocity at the end of any time interval is equal to the velocity at the beginning of the next



Fig. 47

Fig. 48



Fig. 49



interval, we obtain the following expression for the magnitude of acceleration:

$$a = \frac{l_2 - l_1}{\tau^2}. \quad (2.4.1)$$

The acceleration of a body can also be measured by the following easy method. The body is placed in a vessel containing a coloured liquid and having a tap connected to a thin tube with a small hole. When the tap is open, the drops fall from the hole (Fig. 49) in equal intervals of time (which are measured with a stop-watch). As the body moves, the drops mark its position in equal intervals of time. Like in the stroboscopic method, we must measure the lengths  $l_1$  and  $l_2$  of two neighbouring distances between drops. Then the acceleration is calculated by (2.4.1).

---

#### Hometask

Derive formula (2.4.1).

---

## 2.5.

### Free Fall. Acceleration Due to Gravity

The free fall of a body and the motion of a body thrown upwards are interesting examples of rectilinear uniformly accelerated motion.

This type of motion was studied by the Italian astronomer and physicist *Galilei*, who established that these motions are uniformly accelerated. He showed by measurements that the acceleration in these cases is directed vertically downwards and is equal in magnitude to about  $9.8 \text{ m/s}^2$ .

An especially astonishing fact which remained enigmatic for a long time was that *this acceleration is the same for all bodies*.

If we take a steel ball, a football, an unfolded newspaper and a bird's feather, drop all these things from the height of several metres and watch their motion, we shall see that the accelerations of these bodies are different. But this, however, is only due to the fact that the bodies on their way to the



Galileo Galilei (1564-1642), the renowned Italian physicist and astronomer, was the first to use experimental methods for investigating natural phenomena. He discovered the laws of falling bodies and established the law of inertia. He invented the telescope and used it for astronomical observations. He made many important discoveries in this field. As an active supporter of Copernicus' theory of rotation of the Earth, he was subjected twice to inquisition trial and compelled to publicly renounce this theory. According to the legend, Galileo made his "forced" renunciation and then added, "It rotates anyway!"

Earth have to penetrate through air which hampers their motion. If the bodies fall in an evacuated tube, their accelerations would be the same. Such an experiment can be made with the help of a thick-walled glass tube about one metre long, whose one end is sealed and the other is connected to a tap. We place into the tube three different bodies, e.g. a pellet, a cork and a feather, and rapidly turn the tube upside down. All the three bodies fall to the bottom but in different time: first the pellet, then the cork and finally the feather (Fig. 50a). However, the bodies fall like this only when the tube contains air. We have only to pump the air out of the tube with a pump (Fig. 50b), close the tap and turn the tube upside down again (Fig. 50c) to convince ourselves that all the three bodies touch the bottom at the same

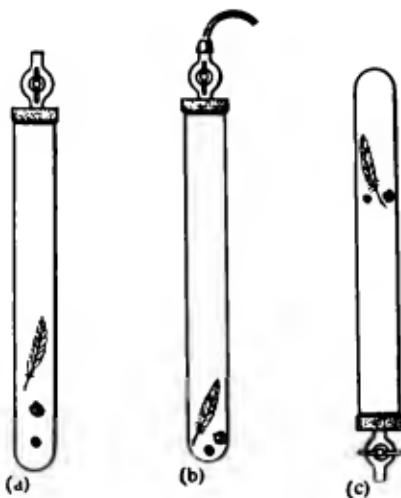


Fig. 50

time. Consequently, in vacuum all bodies fall with the same acceleration.

Such a downward motion in vacuum which is unimpeded by a medium is called the free fall.

In order to distinguish free fall from all other accelerated motions, the free fall acceleration (acceleration due to gravity) is usually denoted by  $\bar{g}$  instead of  $\bar{a}$ . Thus, the vector  $\bar{g}$  is always directed downwards: a body falls with an increasing velocity and moves upwards with a decreasing velocity so that the magnitude of the free fall acceleration  $g = 9.8 \text{ m/s}^2$ . If we direct (as is normally done) the coordinate axis vertically (up or down) and denote it by  $Y$ , the magnitude of the projection  $g_y$  is equal to the magnitude of vector  $\bar{g}$ . The projection is positive or negative depending on whether the  $Y$ -axis is directed downwards or upwards.

---

## Summary

The main problem of mechanics is to determine the position of a body at any instant of time. This problem is solved in a "chain" manner: in order to find the coordinate of a point, we must know its displacement, and to find the displacement, we must know the velocity of motion. Such a velocity-displacement-coordinate chain is used for solving problems in mechanics of uniform rectilinear motion. If the motion is accelerated, we must know the acceleration. Thus, the acceleration-velocity-displacement-coordinate chain is used for such a motion. For uniform as well as uniformly accelerated motion, the "initial conditions" must be specified, viz. the initial coordinates and the initial velocity.

The instantaneous velocity of a body (material point) in uniformly accelerated rectilinear motion changes continuously from one instant of time to another. Therefore, to find the velocity at any instant of time and at any point, we must know the rate of its variation, i.e. the acceleration:

$$\bar{a} = \frac{\bar{v} - \bar{v}_0}{t}.$$

The projection of the velocity of a body onto a chosen coordinate axis at any instant of time  $t$  is calculated by the formula

$$v_x = v_{0x} + a_x t.$$

The coordinate of the body is found from the formula

$$x = x_0 + v_{0x} t + \frac{a_x t^2}{2},$$

while the projection of the displacement  $s_x = x - x_0$  is given by

$$s_x = v_{0x} t + \frac{a_x t^2}{2}.$$

The magnitude of the projection of displacement in a uniformly accelerated motion can also be calculated by the formula

$$s_x = \frac{v_x^2 - v_{0x}^2}{2a_x}.$$

The above formulas can be used for obtaining the expressions for velocity, coordinates and displacements in a uniform rectilinear motion if we put  $a_x = 0$ .

The signs of the projections of vectors  $\vec{v}$ ,  $\vec{v}_0$  and  $\vec{a}$ , as well as the sign of the initial coordinate  $x_0$ , are determined from the conditions of the problem and the direction of the coordinate axis.

---

## 3

## MOTION MORE COMPLEX THAN RECTILINEAR

In nature and in engineering we often deal with motions whose trajectories are curves and not straight lines. Such motions are called *curvilinear*. Planets and artificial satellites move in space along curvilinear trajectories. On the Earth, various vehicles, parts of engines and tools, water in rivers, atmospheric air, etc. are in curvilinear motion.

Problems in mechanics involving a curvilinear motion are more difficult to solve since this motion is more complicated than a rectilinear motion. For a curvilinear motion, we cannot say that only one coordinate of the body is changing. If a motion occurs, for example, in a plane, the two coordinates,  $x$  and  $y$ , are changing (Fig. 51). The direction of motion, i.e. the direction of the velocity vector, also changes all the time. The direction of the acceleration vector may also change. If we add to what has been said above that the magnitudes of velocity and acceleration may also change, it becomes clear that curvilinear motions are much more complicated than rectilinear motions.

### 3.1. Displacement and Velocity in Curvilinear Motion

The direction of the velocity vector in rectilinear motion always coincides with the direction of the displacement. What can we say about the directions of the displacement and velocity in curvilinear motion?

**DISPLACEMENT IS DIRECTED ALONG THE CHORD.** Figure 52 shows a curvilinear trajectory. Suppose that a body moves along it from point  $A$  to point  $B$ . The distance covered by the body is the arc  $AB$ , while the displacement is the vector  $\overline{AB}$  directed along the chord. We cannot now state that the velocity vector is directed in the same way as the displacement vector  $\overline{AB}$ . But we can draw several chords between points  $A$  and  $B$  (Fig. 53) and assume that the body is moving just along these chords. On each chord, the body moves rectilinearly, and the velocity vector is directed along the chord, i.e. along the displacement vector.

**INSTANTANEOUS VELOCITY IS DIRECTED ALONG THE TANGENT.** Let us make our rectilinear segments (chords) still shorter (Fig. 54). As before, the velocity vector on each segment is directed along the

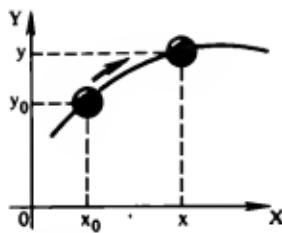


Fig. 51

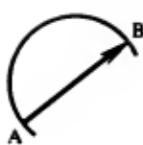


Fig. 52

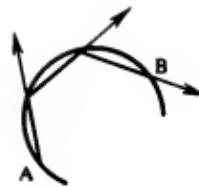


Fig. 53

chord. It can be seen, however, that the broken line consisting of these chords now resembles a smooth curve.

Hence it is clear that if we continue to decrease the lengths of rectilinear segments, we as if contract them to points, and the broken line will become the smooth curve. At each point of this curve, the velocity is directed along the tangent to the curve at this point (Fig. 55).

*At any point of the curvilinear trajectory the velocity of motion of a body is directed along the tangent to the trajectory at this point.*

We can verify that the velocity of a point in curvilinear motion is indeed directed along the tangent while observing, for example, the operation of a grinding lathe (Fig. 56). If we press the ends of a steel rod against a rotating grindstone, red-hot particles detached from the stone will be seen in the form of sparks. These particles fly at the velocity they had at the moment of separation from the stone. We can clearly see that the direction in which the sparks fly always coincides with the tangent to the circle at the point where the rod touches the stone. Splashes from the wheels of a skidding car also fly along the tangent to the circle (Fig. 57).

Thus, the instantaneous velocity of a body at different points of a curvilinear trajectory has, as shown in Fig. 58, different directions. As to the magnitude of the velocity, it can either be the same everywhere (Fig. 58) or change from point to point (Fig. 59).

However even if the magnitude of the velocity in curvilinear motion remains unchanged, the velocity cannot be considered constant. We know that velocity is a vector quantity, and the magnitude and direction are of equivalent importance for vectors. Therefore, a curvilinear motion is always

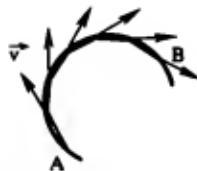


Fig. 54



Fig. 55

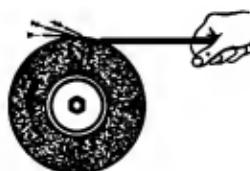


Fig. 56



Fig. 57



Fig. 58

an accelerated motion even if the magnitude of the velocity is constant. We shall limit ourselves to considering this type of motion. It is called *uniform curvilinear motion*. The acceleration in this motion is only due to a change in the direction of the velocity vector. What are the magnitude and direction of this acceleration?

**CURVILINEAR MOTION IS THE MOTION ALONG THE ARCS OF CIRCLES.** Both the magnitude and the direction of acceleration depend on the shape of the curvilinear trajectory. However, we shall not consider each type from the multitude of curvilinear trajectories.

Figure 60 depicts a complex trajectory along which a body is moving. It can be seen that individual parts of the curvilinear trajectory are approximately the arcs of circles shown by dashed lines. For example, segments *KL* and *BM* are the arcs of small circles, while segment *EF* is the arc of the circle having a large radius.

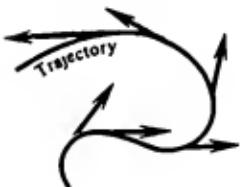


Fig. 59

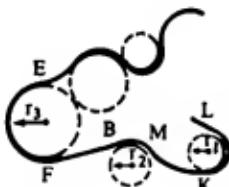


Fig. 60

Thus, the motion along any curvilinear trajectory can be approximately represented as the motion along arcs of some circles. Hence, the problem of determining the acceleration in a curvilinear motion is reduced to finding the acceleration in the uniform motion of the body in a circle.

---

1. What is the direction of the instantaneous velocity in a curvilinear motion?
2. What is the difference in the variation of velocity in curvilinear and rectilinear motions?
3. Can the directions of the velocity and acceleration coincide in a curvilinear motion?
4. Can a body move along a curvilinear trajectory without acceleration?
5. Is it possible for a body to move at a velocity with

a constant magnitude along a polygonal trajectory?

6. What is the relation between curvilinear motion and motion in a circle?

---

## 3.2.

### Acceleration in Uniform Motion of a Body in a Circle

It was shown in Sec. 3.1 that a uniform motion of a body in a circle is an accelerated motion, although the magnitude of the velocity remains unchanged. We now have to find out what magnitude and direction this acceleration has.

**ACCELERATION VECTOR IS DIRECTED TOWARDS THE CENTRE.** The acceleration is defined by the relation

$$\bar{a} = \frac{\bar{v} - \bar{v}_0}{t}. \quad (3.2.1)$$

For the sake of brevity, we denote the difference between two values of velocity (the change in velocity is  $\bar{v} - \bar{v}_0$ ) by  $\Delta\bar{v}$ . Then

$$\bar{a} = \frac{\Delta\bar{v}}{t}. \quad (3.2.2)$$

Obviously, the vector  $\bar{a}$  has the same direction as the vector  $\Delta\bar{v}$  since  $t$  is a scalar quantity.

Let us suppose that a body moves in a circle of radius  $r$  and at a certain instant of time, which will be taken as the initial instant ( $t = 0$ ), it is at a point  $A$  (Fig. 61). The velocity  $\bar{v}_0$  at this point is directed along the tangent. Let us consider another point  $B$ , which is very close to  $A$  and at which the body moving in a circle will be in a certain very small interval of time  $t$ . We shall assume that points  $A$  and  $B$  are so close to each other that the arc  $AB$  cannot be distinguished from the chord  $AB$  (although this cannot be shown in the figure). But in spite of the proximity of these points, the velocity  $\bar{v}$  at point  $B$  still differs from the velocity  $\bar{v}_0$  in direction, although it has the same magnitude ( $v = v_0$ ). We can now find the vector  $\Delta\bar{v} = \bar{v} - \bar{v}_0$  by the method introduced in Sec. 1.4: we translate vector  $\bar{v}$  so that it remains parallel to itself and emerges from the same point  $A$  as vector  $\bar{v}_0$  does (see Fig. 61). Then we connect the heads of the two vectors by a segment of straight line and direct it from  $\bar{v}_0$  to  $\bar{v}$ . We obtain the directed segment, viz. vector  $\Delta\bar{v}$ . It can be seen from the figure that vector  $\Delta\bar{v}$  is directed inside the circle. It can be easily seen that if points  $A$  and  $B$  are infinitely close to each other, this vector is directed towards the centre of the circle. Clearly, the acceleration vector  $\bar{a}$  has the same direction. Thus, the acceleration of a body which moves uniformly in a circle is directed towards the centre of the circle at all points. This acceleration is therefore called *centripetal*.

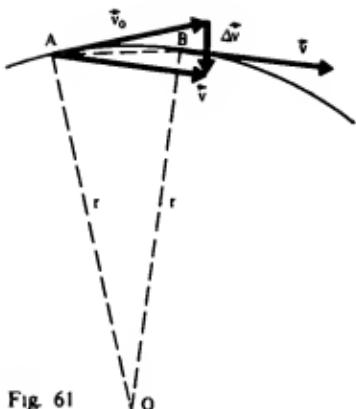


Fig. 61

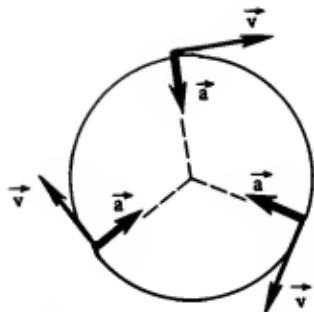


Fig. 62

*A body moving uniformly in a circle has at each point a centripetal acceleration, i.e. the acceleration directed along the radius of the circle towards its centre.*

This peculiarity of the acceleration for a uniform motion in a circle is illustrated in Fig. 62.

**WHAT IS THE MAGNITUDE OF CENTRIPETAL ACCELERATION?** This quantity can be easily found from Fig. 61.

The triangle formed by vectors  $\vec{v}_0$ ,  $\vec{v}$  and  $\Delta\vec{v}$  (see Fig. 61) is isosceles since  $v = v_0$ . The triangle  $OAB$  in this figure is also isosceles because its sides  $OA$  and  $OB$  are the radii of the circle. The angles at the vertices of the two triangles are equal since they are formed by mutually perpendicular sides:  $\vec{v}_0 \perp OA$  and  $\vec{v} \perp OB$ . Therefore, the triangles are similar as isosceles triangles with equal angles at the vertices. The fact that one of the triangles is formed by vectors is immaterial. The similarity of the triangles implies that their similar sides are proportional:

$$\frac{\Delta v}{AB} = \frac{v}{r}.$$

Here  $v$  and  $\Delta v$  are the magnitudes of the velocity and the velocity increment acquired when the body passes from  $A$  to  $B$ ,  $r$  is the radius of the circle and  $AB$  is the chord. It was pointed out earlier, however, that if points  $A$  and  $B$  are infinitely close to each other, the chord  $AB$  is indistinguishable from the arc  $AB$ . The length of the arc  $AB$  is the distance covered by the body having the velocity  $\vec{v}$  of constant magnitude. This distance is  $vt$ . Hence, we can write

$$\frac{\Delta v}{vt} = \frac{v}{r} \quad \text{or} \quad \frac{\Delta v}{t} = \frac{v^2}{r}.$$

Since the time interval  $t$  under consideration is very small,  $\Delta v/t$  is the magnitude of instantaneous acceleration. Consequently,

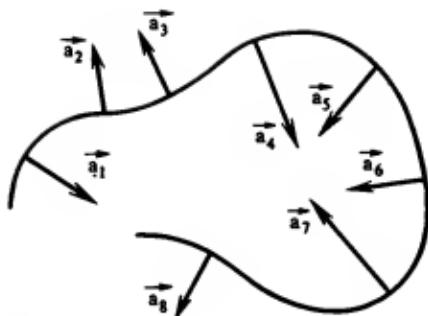


Fig. 63

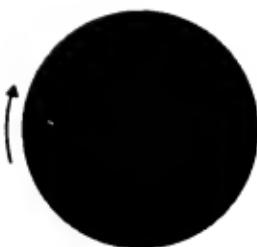


Fig. 64

$$a = \frac{v^2}{r}. \quad (3.2.3)$$

Thus, for a uniform motion in a circle, the centripetal acceleration has the same magnitude at all points of the circle. This acceleration is always directed along the radius towards the centre so that (see Fig. 62) its direction varies from point to point. Therefore, the uniform motion of a body in a circle cannot be treated as uniformly accelerated motion.

It should be recalled that we are interested in the acceleration of a uniform motion in a circle since any curvilinear motion can be represented as the motion along the arcs of circles of various radii.

It can now be stated that if a body moves uniformly along a curvilinear trajectory, its acceleration at any point is directed towards the centre of the circle whose part coincides with the segment of the trajectory containing this point. The magnitude of the acceleration depends on the velocity of the body at this point and on the radius of the corresponding circle. Figure 63 shows a certain complex trajectory and the vectors of centripetal acceleration at its different points.

---

1. What is the direction of acceleration of a body moving in a circle with a velocity having a constant magnitude? What is the magnitude of this acceleration?
2. Will the acceleration of a body moving in a circle with a velocity whose magnitude is varying be directed towards the centre of the circle?
3. Can we assume that the motion in a circle with acceleration of a constant magnitude is a uniformly accelerated motion?
4. Why do car drivers reduce the speed at a steep turn?
5. A speed motorboat towing a sportsman on waterskis moves in a circle. The sportsman can follow the speedboat along the same circle but can also move inside or outside it. What is the relation between the velocities of the sportsman and speedboat in these three cases?

---

### 3.3.

### Period and Frequency of a Body Moving in a Circle

**PERIOD OF REVOLUTION.** The motion of a body in a circle is often characterized not by its velocity but by the time during which it completes one revolution along the circular trajectory. This quantity is called the *period of revolution* of the body and is denoted by  $T$ . For example, the announcements concerning the launching of a satellite usually contain information about the period of its revolution, and the velocity of its orbital motion around the Earth is never mentioned. If, however, the period  $T$  is known, the velocity  $v$  can be easily found. The path traversed by the body during the time interval equal to the period  $T$  is the circumference  $2\pi r$ . Consequently,

$$v = \frac{2\pi r}{T},$$

where  $r$  is the radius of the circle around which the body moves.

Substituting this expression for velocity into formula (3.2.3), we obtain another expression for centripetal acceleration:

$$a = \frac{4\pi^2 r^2}{T^2}. \quad (3.3.1)$$

**FREQUENCY OF REVOLUTION.** The motion of a body in a circle can be also characterized by the number of revolutions made by the body per unit time. This quantity is called the *frequency of revolution* of the body and is denoted by  $n$ . The frequency  $n$  and the period  $T$  of revolution are connected through a very simple relation. For example, if the period of revolution  $T = 0.1$  s, the body completes 10 revolutions in one second. This means that  $n = 1/T$  or, in other words, frequency is the quantity reciprocal to the period. The unit of frequency of revolution is  $1/s$ , or  $s^{-1}$ .

The magnitude  $v$  of velocity of motion in a circle can be expressed in terms of the frequency of revolution  $n$ . Indeed, the velocity is equal to a path traversed by a body in one second. The path traversed by a body during one revolution is  $2\pi r$ . This means that a body completing  $n$  revolutions per second traverses a path equal to  $2\pi r n$  in one second. Consequently,  $v = 2\pi r n$ .

Substituting this expression for  $v$  into (3.2.3), we obtain

$$a = \frac{4\pi^2 r^2 n^2}{r} \quad \text{or} \quad a = 4\pi^2 n^2 r. \quad (3.3.2)$$

While solving problems, use can be made of all the three formulas for centripetal acceleration, viz. (3.2.3), (3.3.1) and (3.3.2).

Formulas (3.3.1) and (3.3.2) show that if the period or frequency of revolution are preset, the magnitude of centripetal acceleration is the larger the longer the radius of the circle. For example, if we consider a rotating wheel (Fig. 64), all the marked points move in circles of different radii but with the same period and the same frequency of revolution. The centripetal acceleration of the points that are further from the axis of rotation is higher.

---

- 1. What is the period of revolution?
- 2. What do we call the frequency of revolution?
- 3. What is the relation between these quantities?
- 4. How can the centripetal acceleration be expressed in terms of period of revolution?
- 5. How can the centripetal acceleration be expressed in terms of frequency of revolution?
- 6. Which points of a rotating wheel (Fig. 64) have a higher and a lower velocity  $v$  of motion in a circle?

---

### Exercise 8

- 1. A grinding wheel whose radius is 10 cm completes one revolution during 0.20 s. Find the velocity of points which are most remote from the axis of rotation.
- 2. A motorcar is moving along a round track of radius 100 m at a velocity of 54 km/hr. Find its centripetal acceleration.
- 3. The period of revolution of the first manned spacecraft "Vostok" was 90 min. The average altitude of the spacecraft above the Earth surface can be assumed to be equal to 320 km, the radius of the Earth is 6400 km. Calculate the orbital velocity of the spacecraft.
- 4. What is the velocity of a moving motorcar if its wheels having the radius of 30 cm complete 10 revolutions per second?
- 5. The Moon moves around the Earth at a distance of 384 000 km, completing one revolution during 27.3 days. Calculate the centripetal acceleration of the Moon.

---

## 3.4. Motion on a Rotating Body

We all live on the surface of the globe which rotates (together with us!) about its axis. However, we do not notice this rotation except when it is manifested in the change of days and nights. But we do not notice the rotation of the Earth only because it rotates very slowly, making only one revolution a day (about  $10^{-5}$  revolution per second).

If, however, a body rotates at a sufficiently high frequency, any other body on its surface is in a very intricate motion. This can be illustrated by a simple experiment: we put a wire ring on a rod (knitting needle, pencil, etc.) and rapidly rotate the rod in the horizontal plane. The ring will slip from the rod. Why does it slip?

Let us consider this experiment in greater detail.

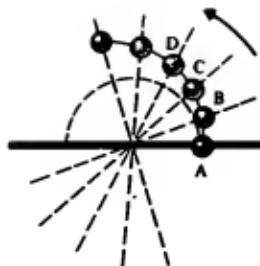


Fig. 65

Suppose that we have a rod on which a small ball with a through hole is put. Let this rod rotate about an axis passing through its middle (Fig. 65). In order to investigate the behaviour of the ball on the rotating rod, we shall consider the rotation of the rod as a sequence of small turns of the rod about the axis.

How does the ball behave during these turns? Suppose that at a certain initial instant of time the ball is at point *A* at a distance *OA* from the axis of rotation *O*. If the ball were fixed on the rod, then during rotation it would move in a circle of radius *OA*, shown in the figure. But it is not fixed. Hence it will move along the velocity vector which, as was shown above, is directed along the tangent to the circle, i.e. normally to *OA*. After a small turn of the rod the ball is at point *B*. Clearly, *OB* is greater than *OA* since the triangle *OAB* is rectangular (Fig. 65) and *OB* is the hypotenuse, while *OA* is the cathetus. During the next small turn, the ball again is displaced perpendicularly to the new position of the rod, viz. *OB*, and gets to point *C*. In the right triangle *OBC*, the side *OC* (hypotenuse) is longer than *OB* (cathetus). The same refers to the triangle *OCD*, etc. Thus, we see that during the rotation of the rod, the ball keeps moving away from axis *O*, sliding along the rod.

If we fix to the rod a system of coordinates and direct one of the axes, say, the *X*-axis, along the rod, in this reference system the ball will move rectilinearly along the *X*-axis. It can be easily seen that in a stationary reference system (relative to the Earth), the trajectory of motion of the rod is much more complex (unwinding spiral). This is one more manifestation of the relativity of motion.

Almost the same happens to children on a disc rotating about the vertical axis (Fig. 66). The children slip off to the disc edges.

## Summary

When a body (material point) is in curvilinear motion, the direction of its velocity vector varies continuously: at each point of the trajectory, it is directed along the tangent at this point. Therefore, even a uniform motion along the curvilinear trajectory, when the magnitude of the velocity remains unchanged, is an accelerated motion.



Fig. 66

The motion of a body in a circle is characterized not only by the velocity  $v$  but also by the period of revolution  $T$  and the frequency of revolution  $n$ . The magnitude of velocity is connected with these quantities through the following relations:

$$v = \frac{2\pi r}{T} \quad \text{and} \quad v = 2\pi r n,$$

where  $r$  is the radius of the circle.

For a body moving uniformly in a circle, the acceleration vector at any point is directed normally to the velocity vector, i.e. to the centre of the circle. For this reason, it is called the centripetal acceleration. Its magnitude is connected with the quantities  $v$ ,  $T$  and  $n$  through the following relations

$$a = \frac{v^2}{r}, \quad a = \frac{4\pi r}{T^2}, \quad a = 4\pi^2 n^2 r.$$

# Fundamentals of Dynamics

## 4 LAWS OF MOTION

### THE MOST IMPORTANT QUESTION IS "WHY"

In the previous part of the book devoted to fundamentals of kinematics, we introduced the quantities used for describing various types of motion observed in the world surrounding us. We have found out that for calculating the velocities of bodies, their displacements and, finally, the coordinates at any instant of time, we must know their accelerations. As a matter of fact, one type of motion differs from another just in acceleration. For example, a uniform rectilinear motion differs from all other types in that its acceleration is equal to zero. A typical feature of a uniformly accelerated motion is that its acceleration is constant in magnitude and direction. The uniform motion in a circle is characterized by an acceleration directed at any point of the circle towards its centre, etc.

The motions of bodies (relative to a chosen reference system) start and cease, are accelerated or slowed down, change their directions, and so on. In all these cases the velocities of moving bodies are changing, i.e. accelerations emerge. Clearly it is very important to be able to find (calculate) acceleration. Without that we can neither solve problems in mechanics nor control a motion. However, to find accelerations we must know why and how they emerge. In physics it is important to find out not only *how* a certain phenomenon occurs but also *why* it occurs in this way and not otherwise. In kinematics, we have learned how the motion occurs (for example, with an acceleration or without it). The question "why do bodies move in this way?" is answered in the main part of mechanics, viz. *dynamics*.

### 4.1. Bodies and Surroundings. Newton's First Law

In order to find the reason behind the emergence of acceleration, we must turn to experiments or observations. But first let us determine the conditions under which a body moves without an acceleration, i.e. when its velocity does not vary with time.

Any body, in motion or at rest, is not solitary in the world. There are many other bodies around it—close and remote, large and small, stationary and moving. It is natural to assume that some of these bodies, if not all of them, influence the body under consideration in a certain way, affecting its state of motion. We cannot distinguish beforehand the surrounding bodies which affect this state considerably from those whose influence is insignificant. This should be investigated in each separate case.

Let us first consider a body at rest. The acceleration of this body is equal to zero as well as its velocity.

Figure 67 depicts a ball suspended from a rubber cord. The ball is at rest relative to the Earth. There are many different bodies around the ball: the cord from which it is suspended, the walls of the room, the objects in this and neighbouring rooms and, of course, the Earth. Obviously, these bodies act on the ball not in the same way. If, for example, we rearrange the furniture in the room, this will not produce any noticeable effect on the ball. But if we cut the cord (Fig. 68), the ball will immediately start to fall with an acceleration.

It is well known that all bodies fall down just under the action of the Earth. But until the cord is cut, the ball is still at rest. This simple experiment shows that of all the bodies surrounding the ball, two bodies have a noticeable effect: the rubber cord and the Earth. Their joint effect ensures that the ball is at rest. We had only to eliminate one of these bodies (the cord), and the state of rest was violated.

If we could remove ... the Earth (retaining the action of the stretched cord), this would also disturb the state of rest of the ball. It would start to move in the opposite direction (upwards).

Thus, we can draw the conclusion that the actions on the ball of the two bodies—the cord and the Earth—cancel or balance each other.

When the effects of two or more bodies are said to compensate each other, this means that the result of their joint influence is the same as if these bodies were absent.

The example considered above, as well as many other similar examples, leads us to the following conclusion: *a body is at rest if the actions of other bodies on it compensate each other.*

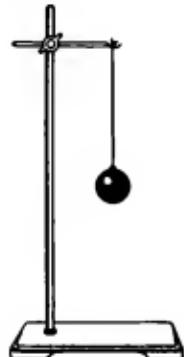


Fig. 67

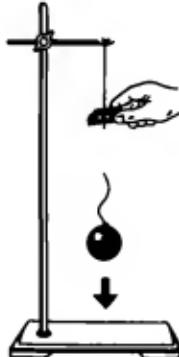


Fig. 68

## SPORT



Fig. 69

We know, however, that motion as well as the state of rest is relative. A body which is at rest relative to some reference system can move relative to some other systems. Let us consider, for example, a puck lying on the ice of a hockey field (Fig. 69). The puck is at rest relative to the ice (the Earth) since the action of the Earth on it is balanced by the action of the ice. But for a hockey-player moving past the puck rectilinearly and uniformly, the puck is also in uniform rectilinear motion. Thus, the same body (the puck) is at rest in one reference frame (fixed to the Earth) and is moving rectilinearly and uniformly relative to the other reference system (fixed to the hockey-player).

The hockey-player strikes the puck by the stick. As a result of the very short action of the stick, the puck is set in motion, having acquired a certain velocity. It is remarkable that after the impact, when the action of the stick on the puck has ceased, the puck continues its motion. Meanwhile, the effect of other bodies on the puck after the impact has remained the same: as before, the action of the Earth is balanced by the action of the ice, and the stick does not produce any effect on the motion of the puck any longer. However, after the impact the puck is moving along the straight line at an almost constant velocity which it acquired at the moment of impact. True, in the long run the puck stops, but we know from experience that the smoother the ice and the puck surface, the longer the puck moves. Hence we can guess that if it were possible to eliminate the action of the ice on the puck, which is called friction, the puck would continue to move relative to the Earth at a constant velocity for an infinitely long time.

If, however, a hockey-player moved alongside with the uniformly sliding puck at the same velocity, the puck would be at rest relative to him (or to the reference system associated with him). In this case too the same body is in a uniform rectilinear motion relative to one reference system (the Earth) and is at rest relative to the other system (hockey-player).

**NEWTON'S FIRST LAW.** This and many other similar examples lead us to one of the fundamental laws of mechanics, called the first law of motion, or *Newton's first law*.

There exist such reference systems relative to which a body in translatory

motion preserves its velocity unless other bodies are acting on it (or the action of other bodies is not balanced).

The property of preserving the velocity of a body (in particular, the state of rest) when external effects on the body are balanced is called *inertia*. Therefore, Newton's first law is often called the *law of inertia*. The common expression "inertial motion (coasting)" just means that a body moves at a constant velocity (i.e. is in a uniform rectilinear motion) when the actions of other bodies on it are balanced. In the same way, we can speak about the "state of rest by virtue of inertia".

**INERTIAL REFERENCE SYSTEMS.** Reference systems mentioned in Newton's first law, i.e. the systems relative to which the body is at rest or in uniform rectilinear motion when external effects are balanced, are called *inertial reference systems*. In the examples considered above, the inertial systems were the system fixed to the Earth and the system associated with the hockey-player moving uniformly in a straight line relative to the Earth. We can verify whether a reference system is inertial or not only in experiment. Since experiments show that any reference system fixed to the Earth can be approximately considered as inertial, a system associated with any body which moves uniformly in a straight line is also inertial relative to the Earth.

The law of inertia is not as obvious as it may seem at first sight. The discovery of this law put an end to an old delusion. Before this law was discovered, it was assumed for centuries that in the absence of external influences (or, which is the same, when all these influences are balanced) a body can be only in a state of rest which was assumed to be a natural state of the body. It was believed that the motion of a body at a constant velocity requires another body acting permanently on it. This may seem to be confirmed by everyday experience: in order to make a carriage move at a constant velocity, a horse must pull it all the time. To move a table over the floor, it must be continuously pushed or pulled.

The great Italian scientist GALILEI was the first to find that this statement is erroneous and that in the absence of external influence a body can not only be at rest but also move uniformly in a straight line. Consequently, uniform rectilinear motion is a "natural" state of bodies like a state of rest. We must push or pull the table to set it in motion since the floor not only balances the action of the Earth on the moving table but also creates an additional effect called friction. The action of those who pull or push the table is needed for balancing friction. Galilei drew the conclusion that if friction were absent, a body (table) set in motion would continue to move at a constant velocity without any effect from outside.

The brilliant English physicist ISAAC NEWTON generalized Galilei's conclusions and included them in fundamental laws of motion.

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?            1. Rowers who strive to move a boat against the stream are unable to cope with their job and the boat remains at rest relative to the bank. The actions of which forces are balanced in this case?

2. What is the meaning of inertia?
3. Formulate Newton's first law.
4. A table tennis ball is lying on a table. The table was pushed and the ball started to move. Indicate the reference body relative to which the law of inertia is valid and the one relative to which this law does not hold.
5. Figure 1 illustrates a translatory motion in which a body (suitcase) is moving not in a straight line. Is Newton's first law violated in this case?

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#### Hometask

1. Give examples of bodies in a state of rest. The action of which bodies is compensated in these cases?
2. Give examples of bodies moving uniformly in a straight line. Indicate the bodies whose actions are mutually compensated.

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## 4.2.

### Interaction of Bodies. Acceleration of Bodies as a Result of Their Interaction

According to Newton's first law, a body moves without an acceleration, i.e. uniformly and in a straight line relative to an inertial reference system, if no other bodies are acting on it or if the actions are present but they are compensated.

Let us now determine the conditions under which bodies move with an acceleration. Experiments show that when a body is moving with an acceleration, we can always indicate another body or several bodies whose influence causes this acceleration. For example, falling bodies move with an acceleration. The body causing their acceleration is the Earth. A puck lying on ice has changed its velocity as a result of an impact. The body imparting an acceleration to the puck is the stick.

Let us bring a magnetized steel rod (a magnet) to an iron ball. The ball which has been at rest starts to move. It acquires an acceleration (Fig. 70) due to the action of the magnet. Until the magnet stops acting, the ball will move with an acceleration, continuously increasing its velocity.

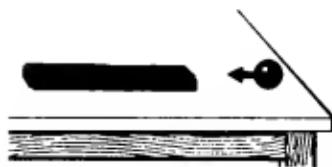


Fig. 70

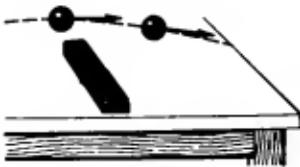


Fig. 71

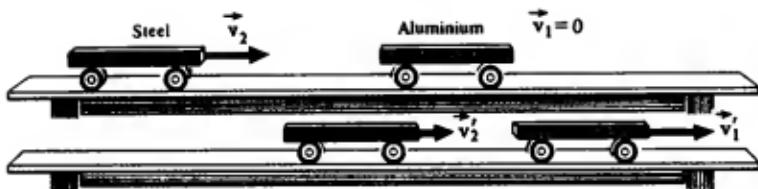


Fig. 72

If we bring the magnet close to the moving ball as is shown in Fig. 71, the *direction* of the ball velocity will change: the trajectory of the ball will be curved. As we know, this means that the ball has acquired a centripetal acceleration. This experiment demonstrates once more that the influence of an external body just causes a change in motion rather than the motion itself. The ball was moving before we brought the magnet to it, was it not?

Thus, *the cause of acceleration of a body is the influence of other bodies on it.*

What determines the magnitude and direction of acceleration acquired by a body due to the influence of another body? To answer this question, we again turn to an experiment.

**INTERACTION OF BODIES.** In the simplest case, two bodies should take part in the experiment, viz. the influencing body and the body that experiences this influence.

Actually, the two bodies have, so to say, "equal rights". Each of them influences the other and at the same time experiences a reaction. For example, when a football player runs into another player, they both change their velocities.

Generally, every time a body *A* acquires an acceleration due to the action of a body *B*, the body *B* also acquires an acceleration. It is said that *interaction of bodies* takes place and both of them acquire accelerations. What accelerations are these?

A large number of experiments carried out with various bodies revealed that *accelerations of two interacting bodies have opposite directions*. Besides, the *ratio of magnitudes of accelerations* of these bodies is always the same. This ratio is completely independent of the nature of interaction of the bodies. It can be a collision between two bodies or the interaction of the same bodies connected through a spring, thread or wire. Finally, the bodies may interact without touching each other in the same way as planets interact with the Sun, the Moon with the Earth or a magnet with a piece of iron. The magnitudes of accelerations of each body can be quite different for different types of interaction, but their ratio remains unchanged.

If, for example, we took two carts of the same size but one made of aluminium and the other of steel (Fig. 72) and made them collide, they would change their velocity during the collision and get accelerated. The measurements would show that the magnitude of the acceleration  $\bar{a}_1$  of the aluminium cart is three times larger than the acceleration  $\bar{a}_2$  of the steel cart

irrespective of the velocities the carts had before the collision:

$$\frac{a_1}{a_2} = 3.$$

The accelerations of the carts have opposite directions.

It is very difficult to measure the accelerations of the carts during the collision, since the time of the collision is very short. It is much easier to carry out an experiment in which the interacting bodies move uniformly in a circle and to measure the centripetal accelerations of these bodies.

The schematic diagram of such an experiment is shown in Fig. 73. Two cylinders, one of steel and the other of aluminium, having the same size and with holes drilled along their axes, are put on a rod along which they can slide with a small friction.

We fix the rod with the cylinders to centrifugal machine and set it in rotation. The cylinders will immediately slide towards the ends of the rod (see Sec. 3.3). In this experiment, the cylinders do not interact with each other.

We then tie the cylinders together with a thin thread and rotate the centrifuge again. The cylinders are now interacting with each other through the thread.

At certain distances of the cylinders from the axis of rotation of the rod, they no longer slide from the rod and move in circles. The radii  $r_1$  and  $r_2$  of these circles are the distances of the cylinders from the axis of rotation. As we know, a body moves in a circle with a centripetal acceleration directed towards the centre of rotation and equal to  $4\pi^2 n^2 r$ , where  $n$  is the frequency of revolution and  $r$  is the radius of the circle. The ratio of the accelerations of the aluminium and steel cylinders is hence

$$\frac{a_1}{a_2} = \frac{4\pi^2 n^2 r_1}{4\pi^2 n^2 r_2} = \frac{r_1}{r_2}.$$

Measuring the radii  $r_1$  and  $r_2$ , we see that the radius  $r_1$  for the aluminium cylinder is three times the radius  $r_2$  of the circle in which the steel cylinder rotates. Therefore, the ratio of the accelerations of the cylinders is equal to 3.

We can change the length of the thread connecting the cylinder or the number of revolutions of the rod per unit time. This will change the accelerations of the cylinders. However, the experiment shows that for any type of interaction of these two bodies, the ratio of the magnitudes of their accelerations remains unchanged.

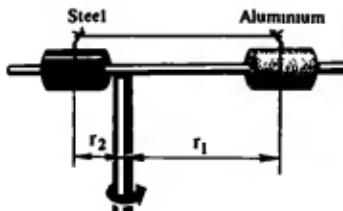


Fig. 73

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9

1. What is the cause of acceleration?
2. What can we say about the accelerations of two interacting bodies?
3. As a result of interaction of two bodies, the velocity of one of them has increased. What is the change in the velocity of the other body?

---

### Exercise 9

1. Find the velocity of the aluminium cart mentioned in this section after its collision with the steel cart, if the initial velocity of the steel cart is 4 m/s and its velocity after the collision is 2 m/s. The aluminium cart was at rest before the collision.
2. The aluminium and steel cylinders used in the experiment described in this section are connected by a 8-cm thread. At what distance from the centre of the rod will each cylinder be?
3. In the same experiment, the cylinders were tied by a thread of a different length. It turned out that during rotation of the rod, the aluminium cylinder was at a distance of 9 cm from its centre. What was the length of the thread?

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### Hometask

Give examples showing that interaction of bodies is the cause of the change in motion (velocity) of the bodies rather than of the motion itself.

---

## 4.3. Inertia of Bodies

Experiments considered in Sec. 4.2 have shown that the ratio of magnitudes of the accelerations acquired by two bodies during their interaction does not depend on the nature of the interaction and is determined only by the bodies themselves. Consequently, each body has a certain property which determines the ratio of the magnitude of its acceleration to the magnitude of the acceleration of the body with which it interacts.

What kind of property is this?

When a body is moving without acceleration, i.e. at a constant velocity, it moves by inertia. As a result of interaction between bodies, the velocity of each of them changes. The experiments considered in Sec. 4.2 show that the accelerations of interacting bodies are different. The fact that the acceleration of one body is smaller in magnitude than that of the other implies that during the same time of interaction, the velocity of one of the bodies changes

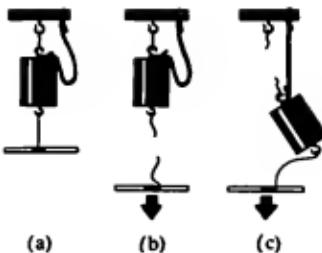


Fig. 74

to a smaller extent than the velocity of the other. It should be recalled that acceleration is the ratio of the change in velocity to the time interval  $t$  during which this change occurs:

$$a = \frac{\bar{v} - \bar{v}_0}{t}.$$

Therefore, the smaller the acceleration of a body, the smaller is the change in its velocity over a given time  $t$ .

The body which changes its velocity to a smaller extent as a result of interaction is said to be more inert than the other body. If the body did not change its velocity at all, it would move by inertia, i.e. uniformly and in a straight line, would it not?

*Inertia* is the property inherent in all bodies. *It consists in that it takes some time for a body to change its velocity. The longer the time required for changing velocity by a given value, the more inertia the body has.* Of two interacting bodies, the one which changes its velocity slower is more inert.

The following example clearly illustrates the manifestation of inertia of bodies and the role of the time during which one body is acting on another.

A cylinder is suspended from a thin thread (Fig. 74a). Another thread is tied to the bottom of the cylinder. If we abruptly jerk the lower thread, it breaks, while the cylinder continues to hang on the upper thread (Fig. 74b). If, however, we pull the lower thread slowly instead of jerking, the upper thread breaks and the cylinder falls (Fig. 74c). This is explained as follows. If we jerk the lower thread abruptly, the time of its action on the cylinder is so short that the cylinder cannot considerably increase its velocity (has no time to gain speed) and be noticeably displaced downwards. Hence the upper thread remains intact. On the other hand, the lower thread is less inert and acquires a considerable velocity during the jerk. Its displacement turns out to be sufficient for a breakdown. If, however, we pull the lower thread slowly, it acts on the cylinder for a long time during which it acquires such a velocity that its displacement is enough to rupture the upper thread which has been already stretched.

?

1. Can the velocity of a body change instantaneously?
2. Which property of bodies is called inertia?

---

### Hometask

Give examples showing that the velocities of two interacting bodies change simultaneously.

---

## 4.4. Mass of Bodies

Inertia, which is inherent in every body, is one of the most important properties since it determines the acceleration acquired by a body as a result of its interaction with other bodies.

Physics studies the properties of bodies which can be characterized by a certain quantity. The property called inertia is also characterized by a definite quantity. This quantity is the *mass* of the body.

Of two interacting bodies, the more inert body, i.e. the one acquiring the smaller acceleration, has the larger mass. If we denote the masses of the interacting bodies by  $m_1$  and  $m_2$ , we can write

$$\frac{a_1}{a_2} = \frac{m_2}{m_1} \quad (4.4.1)$$

*The ratio of the magnitudes of accelerations of two interacting bodies is equal to the inverse ratio of their masses.*

It was shown, for example, that the ratio of the accelerations of an aluminium and a steel cylinder is equal to three. This is due to the fact that the mass of the aluminium cylinder is equal to one third the mass of the steel cylinder.

Thus, we now know how to find the ratio of the masses of two bodies. For this purpose, we must measure their accelerations due to interaction. But how can we determine the mass of each body? In order to find the number expressing the mass of an individual body, we must first choose a body whose mass will be conditionally taken as a unit or the *standard mass*. Then we must carry out an experiment in which the mass of the body being determined somehow interacts with the standard mass (see Fig. 73). As a result of this interaction, both the body and the standard will acquire accelerations which can be determined experimentally. Then we can write the relation

$$\frac{a_{st}}{a_b} = \frac{m_b}{m_{st}}$$

or

$$m_b = \frac{a_{st}}{a_b} m_{st}, \quad (4.4.2)$$

where  $m_b$  and  $a_b$  are the mass and the magnitude of acceleration of the body while  $m_{st}$  and  $a_{st}$  are the mass and the magnitude of acceleration of the

standard. But the mass of the standard is equal to unity, and hence

$$m_b = \frac{a_{st}}{a_b} \text{ units of mass.}$$

*The mass of a body is the quantity which characterizes its inertia. It determines the ratio of the magnitude of acceleration of the standard mass to the magnitude of acceleration of the body in their interaction.*

It should be recalled (see *Junior Physics*, Sec. 22) that the standard mass is a specially made cylinder of platinum-iridium alloy. The mass of this cylinder is taken as the international unit of mass – kilogram (kg). To a sufficient degree of accuracy, we can assume that one litre of pure water at 15°C has the mass of one kilogram.

Along with such quantities as length and time, mass is included in basic SI units.

It should not be thought that each time when the mass of a body has to be measured the body is made to interact with the standard mass and the accelerations of the body and the standard are determined. Of course, this method is practically inconvenient. There exists another method of measuring mass, viz. *weighing*, which is commonly used. This method of measuring mass is well known from the course of *Junior Physics*. However, in certain cases, measurement of accelerations due to interaction is the only possible method of determining the mass. For example, the mass of planets, stars and other celestial bodies cannot be determined by weighing. Very small masses like the masses of atoms and particles constituting them cannot be measured with the help of a balance.

The mass of a body expresses its intrinsic property (inertia) which does not depend on the interactions in which the body takes part or on its motion. No matter where a body is and how it moves, its mass remains the same.

**MASSES ARE ADDED.** Let us carry out one more experiment (Fig. 75) in order to investigate an interesting and important property of mass. We connect two identical aluminium cylinders and repeat the experiment with a centrifuge (see Sec. 4.2). The steel cylinder is now interacting not with one but with two connected aluminium cylinders. The experiment shows that the ratio of accelerations of two connected aluminium cylinders and the steel cylinder is 3/2 and not 3. This means that the mass of two identical cylinders connected together to form as if a single body is twice as large as the mass of one of them. Consequently, *when two or more bodies are combined into one, their masses are added*.

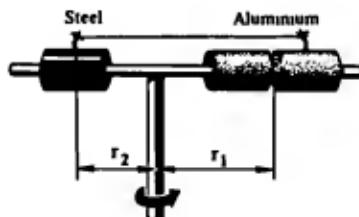


Fig. 75

ONCE MORE ABOUT THE THEORY OF RELATIVITY. It was mentioned above that the mass of a body does not depend on the type of its motion. This, however, is not quite so. In Sec. 1.8 we have found out that according to the theory of relativity, time passes differently in different reference systems moving relative to each other. This leads to many wonderful consequences. It turns out, in particular, that the mass of a body changes during its motion. Suppose that the mass of a body at rest is  $m_0$ . If we could measure with the help, for example, of a centrifuge the mass of this body when it is moving at a velocity  $v$ , it would turn out that it is not equal to  $m_0$  and is given by

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}},$$

where  $c$  is the velocity of light. Consequently, the mass of the body has increased. However, this increase in mass becomes noticeable only at velocities close to the velocity of light ( $c = 3 \times 10^8$  m/s). Ordinary bodies never move at such velocities. The fastest body encountered on the Earth is the Earth itself which rotates around the Sun at a velocity of 30 km/s. At such velocities the mass can be treated as a constant quantity.

?

1. Which quantity characterizes the inertness of a body?
2. What is the relation between the masses of bodies and the magnitudes of accelerations acquired by them as a result of interaction?
3. How can the mass of an individual body be determined?
4. What is taken as the standard of mass?

### EXAMPLE OF SOLVING A PROBLEM

Compare the masses of the Moon and the Earth if the radii of the orbits of the Moon and of the centre of the Earth are known.

*Solution.* It is usually assumed that the Moon (under the action of the Earth) rotates about the Earth so that the centre of the Earth is as if the fixed centre of the lunar orbit. This, however, is impossible since both interacting bodies acquire accelerations as a result of interaction. Actually, the Moon also influences the Earth, making it move in a circle and imparting a centripetal acceleration to it. But around which centre is the Earth moving?

Astronomical observations have shown that the Moon rotates not about the centre of the Earth but about a certain point  $P$  (Fig. 76) which is 4700 km away from the centre of the Earth. The centre of the Earth also moves about this very point  $P$  (Fig. 77). The centre of the Earth is moving in a circle of radius  $r_E \approx 4700$  km, while the centre of the Moon is moving in a circle of radius  $r_M \approx 380\,000$  km. Hence the Earth and the Moon behave in the same way as the aluminium and steel cylinders considered in Sec. 4.2. It was shown in the example with the cylinders that the ratio of magnitudes

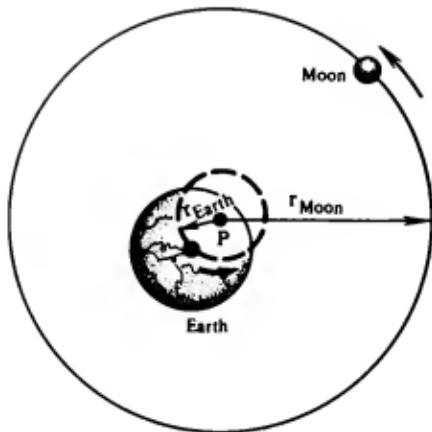


Fig. 76

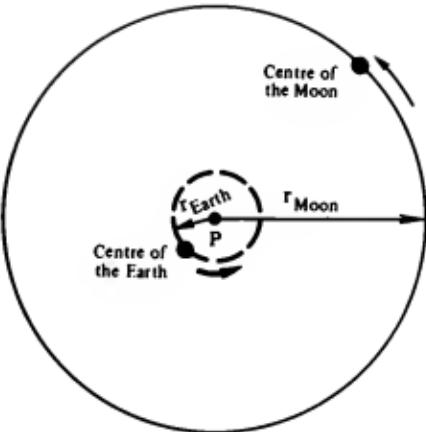


Fig. 77

of the centripetal accelerations imparted to the cylinders is equal to the ratio of radii of the circles in which they are moving. Hence, we can write

$$\frac{a_M}{a_E} = \frac{r_M}{r_E}.$$

But as we know, the ratio of magnitudes of the accelerations of interacting bodies is equal to the inverse ratio of their masses. Hence

$$\frac{r_M}{r_E} = \frac{m_E}{m_M}.$$

Since  $r_M \approx 380\,000$  km and  $r_E \approx 4700$  km, we obtain

$$\frac{m_E}{m_M} = \frac{380\,000 \text{ km}}{4700 \text{ km}} \approx 81.$$

### Exercise 10

1. A cart is moving over a horizontal surface at a velocity of 50 cm/s. It is hit by another cart which is moving in the same direction at a velocity of 150 cm/s. After collision, the carts continue to move in the same direction at the same velocity of 100 cm/s. Find the ratio of the masses of the carts.
2. A cart is moving over a horizontal surface at a velocity of 30 cm/s and collides with a cart of the same mass which is at rest. As a result of the collision, the moving cart comes to a halt. What is the velocity acquired by the other cart?

## 4.5.

### Force

It should be recollected that our aim is to calculate accelerations of moving bodies. Without this, the basic problem of mechanics cannot be solved.

It was shown in previous sections that when a certain body 1 with mass  $m_1$  acquires an acceleration  $\bar{a}_1$ , this is due to the fact that some other body 2 with mass  $m_2$  is exerting an influence on it. The second body, in turn, also acquires an acceleration  $\bar{a}_2$  such that

$$a_1 = \frac{m_2}{m_1} a_2.$$

It may seem that according to this formula the motion of only one body (we shall call it the body being accelerated) cannot be studied and its acceleration cannot be calculated. We must know the mass and acceleration of another body (which will be called accelerating body).

**FORCE IS THE CAUSE OF ACCELERATION.** Usually, we are interested in the motion of just one body (being accelerated) and not the body or bodies which influence it by imparting an acceleration. For example, when a shell is fired from a barrel, it interacts with the Earth and the air through which it flies. Both the Earth and the air impart accelerations to the shell and simultaneously acquire certain accelerations themselves. For the gunner, however, it is important to know only the acceleration of the shell. Why does he need to know the masses and accelerations of the Earth and air? Therefore, the acceleration of only one body is usually calculated, viz. the body whose motion is being studied. *The effect of the other body causing an acceleration is briefly called the force acting on the body being accelerated.* Thus, instead of saying that the acceleration of a body is caused by the influence of another body on it, it is said that the *acceleration is caused by a force applied to the body* (or acting on it).

Let us consider the following example. Suppose that one end of a spring of length  $l$  is fixed (Fig. 78, top). A bar connected to the other end of the spring remains at rest. Let us stretch the spring (without a bar) by  $\Delta l$  (Fig. 78, bottom) and again fix the bar to its free end. When we release the spring, the bar will move with an acceleration. Obviously, the acceleration is caused by the interaction of the bar and the spring. Now we can say that it is caused by the *force due to stretching the spring*. This force is called the *elastic force*.

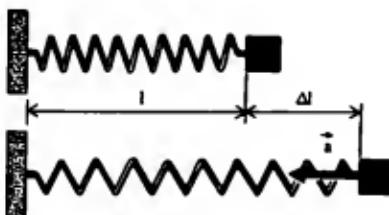


Fig. 78

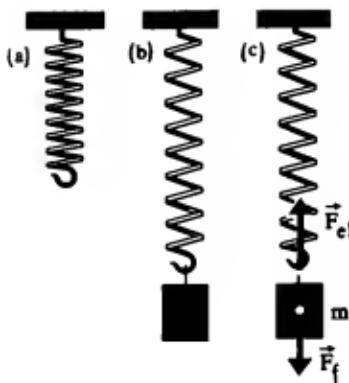


Fig. 79

Since the elastic force emerges only as a result of stretching the spring (no force appears if there is no stretching!), it depends only on the extent to which the spring is stretched. Here is another example. It is well known that a freely falling body and a body thrown upwards move with an acceleration. This acceleration is caused by the interaction of a body with the Earth. But now we can say that the acceleration is caused by the *force exerted on the body by the Earth*. This force is called the *force of gravity*.

**FORCE IS A PHYSICAL QUANTITY.** It appears that the elastic force and the force of gravity are entirely different. They are unlike if only due to the fact that the spring is acting on a body in contact while the Earth exerts force without touching a body. However, these actions are similar in that they impart accelerations to bodies.

One force may impart a large acceleration to a body, while another may cause only a small acceleration. Hence, force is a physical quantity that can be expressed by a number, but not only by it!

Let us consider Fig. 79b. It shows a load which is suspended to a spring and is at rest in this position.

The load is acted upon by the force of gravity due to the presence of the Earth and the elastic force caused by stretching the spring (cf. Fig. 79a). Each of these forces can impart an acceleration to the body. The force of gravity (if it were acting alone) would impart the acceleration  $\bar{g}$  to the body. On the other hand, if the elastic force were the only force acting on the load, it would impart to it a certain acceleration  $\bar{a}$ . But the load is at rest. This means that accelerations  $\bar{g}$  and  $\bar{a}$  are equal in magnitude and opposite in direction:  $\bar{g} = -\bar{a}$ . Consequently, their sum is equal to zero.

What can be said about forces? Clearly, if two forces are applied to a body and the body has no acceleration, the sum of these forces is zero. This means that like the accelerations, these forces are equal in magnitude and have opposite directions.

Hence it follows that a *force is defined not only by a number but also by a direction, i.e. force is a vector*. For this reason, the forces  $F_{el}$  and  $F_g$  are depicted in Fig. 79 by arrows of the same length and opposite directions.



Sir Isaac Newton (1643–1727) was one of the greatest physicists and mathematicians of all times. He formulated the general laws of mechanical motion, discovered the law of universal gravitation, and formulated the basic principles of differential and integral calculus. Newton made significant contributions in the field of optics. Basically, all these discoveries and investigations were made by Newton when he was about 25 years old. These results were published much later in the form of two books, the magnificent treatise entitled "Philosophiae naturalis principia mathematica" (1686), and "Opticks" (1704).

Thus, the word "force" corresponds to a physical quantity that expresses the action of one body on another.

What kind of quantity is it? What is its magnitude? And above all, how is it related to acceleration? The answers to these questions are given by the most important law of motion, viz. Newton's second law.

## 4.6. Newton's Second Law

**FORCE AND ACCELERATION.** To find the relation between force and acceleration, we must again turn to an experiment. In this experiment, the *same force* should impart acceleration to different bodies, i.e. the bodies having different masses, so that the acceleration of these bodies could be measured.

For this experiment, we must choose a body that acts on all other bodies with the same force. It can be a stretched or compressed spring which generates an elastic force. The remarkable feature of this force which distinguishes it from all other forces is that it depends *only* on the deformation of the spring and does not depend on the body to which it is attached.<sup>11</sup> Thus, a spring stretched to a certain length acts on *any* body attached to it with the same force, viz. the elastic force of the spring.

Since the force is the same, a certain quantity associated with acceleration should be the same for all bodies. In this experiment, we shall find out what is this quantity.

For example, we can carry out such a simple experiment. We fix one end of a spring to a cart of a known mass  $m$ . To its other end, we tie a cord with a load thrown over through a pulley (Fig. 80a). Owing to attraction to the Earth, the load moves downwards and stretches the spring. The spring is

<sup>11</sup> Experiments show that there are no other forces in nature that would possess this property.

stretched by a certain length  $\Delta l$ , exerts an elastic force on the cart and imparts to it an acceleration. This acceleration can be measured, for example, by the stroboscopic method (see Sec. 2.4). Suppose that this acceleration is equal to  $a$ .

Let us repeat this experiment but instead of one cart take two identical carts joined together (Fig. 80b), so that their total mass is  $2m$ . We must measure the acceleration of this "train" for the *same elongation*  $\Delta l$  of the spring, since the force should remain the same. In order to get the same elongation of the spring as in the first experiment, we should choose and suspend another load from the cord. The experiment will show that for the same elongation  $\Delta l$  of the spring, the acceleration of the two connected carts is  $a/2$ . If we make a train of three, four, etc. carts, for the same elongation of the spring the acceleration will be equal to  $1/3$ ,  $1/4$ , etc. of the acceleration of one cart. It turns out that if the mass of the cart is increased a certain number of times, the acceleration imparted to it by the same force decreases in the same proportion. This means that the *product of the mass of the cart by its acceleration remains unchanged*.

It is easier to carry out this experiment by imparting centripetal accelerations to bodies of different mass. Let us again use the centrifugal machine.

A body  $M$  in the form of an aluminium cylinder with a hole drilled along its axis is put on the rod of a centrifuge (Fig. 81a). We connect one end of a spring to the cylinder and fix its other end to the frame of the centrifugal machine at point  $A$ . Then we set the machine in rotation. As was shown in Sec. 4.1, the cylinder  $M$  will slide along the rod, moving apart from point  $A$  and thus stretching the spring. If the spring were absent, the cylinder would come to a halt at point  $B$ . But due to the elastic force of the stretched spring, the cylinder will be shifted from the axis of rotation (by a distance  $\Delta l$ ) and will then move in a circle of radius  $r$  (Fig. 81b). The centripetal acceleration of cylinder  $M$  is directed along the radius to the centre of rotation. The axis of the spring is also directed along the radius.

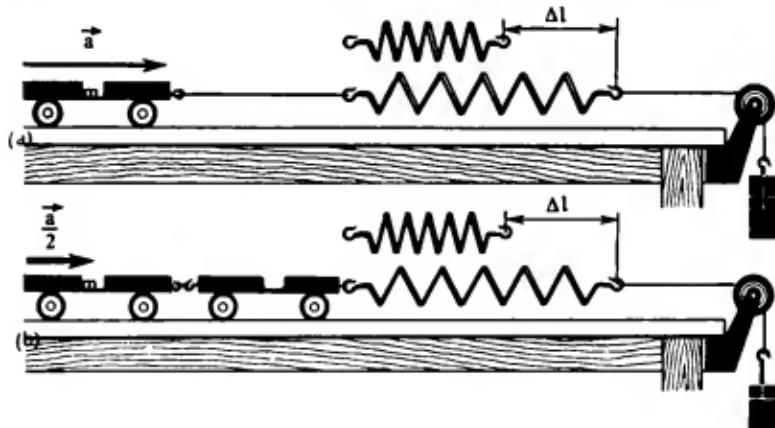


Fig. 80

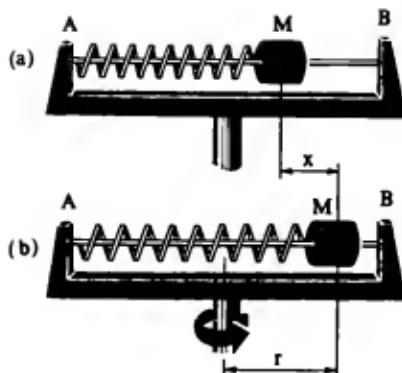


Fig. 81

Consequently, the acceleration of the cylinder  $M$  is directed along the axis of the spring parallel to the elastic force. Clearly, it is this force that imparts the centripetal acceleration to the body.

The magnitude of centripetal acceleration  $\ddot{a}$  is given by

$$a = 4\pi^2 n^2 r,$$

where  $n$  is the frequency of revolution and  $r$  is the radius of the circle in which the body moves.

By measuring  $n$  and  $r$ , we find the magnitude of the acceleration  $\ddot{a}$ .

Figure 82 shows an instrument used in school laboratories for demonstrating this experiment.

Let us replace the aluminium cylinder by a steel one of the same size. As we know, its mass is three times the mass of the aluminium cylinder. Let us set the centrifuge in rotation again and choose such a frequency of revolution that the elongation of the spring is the same as in the first experiment. Then the force acting on the cylinder is also the same. The experiment shows that the acceleration of the steel cylinder is less than the acceleration of the aluminium one by a factor of three.

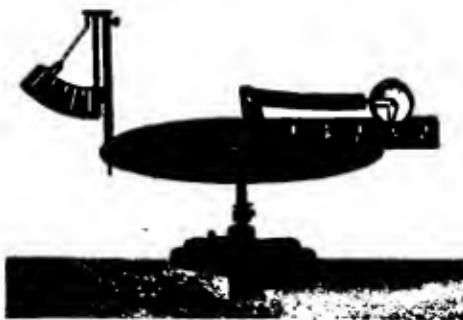


Fig. 82

NEWTON'S SECOND LAW. We can carry out the experiment described above with a large number of bodies of different mass. As in the experiments with the carts, we shall see that accelerations of different bodies are different but the product of the mass of a body and its acceleration is the same for all the bodies. Thus, we find from experiment that if the same force is acting on different bodies, the product of the mass of a body and its acceleration also remains the same.

This gave grounds to Newton to state that the force is equal to the product of the mass of a body and its acceleration and to formulate the most important law of mechanics which is called *Newton's second law*:

The force acting on a body is equal to the product of the mass of the body and the acceleration imparted by this force.

If we denote the force by  $F$ , the analytic form of Newton's second law is  $F = ma$ . This formula refers to the *magnitude* of force. But since acceleration is a vector and mass is a scalar, the force is a vector quantity and vectors  $F$  and  $\vec{a}$  have the same directions. Therefore we can write the formula expressing Newton's second law in the following form:

$$\underline{\underline{F}} = m\underline{\underline{a}}. \quad (4.6.1)$$

From this formula, we can obtain the expression for  $\vec{a}$ :

$$\underline{\underline{a}} = \frac{\underline{\underline{F}}}{m}, \quad (4.6.2)$$

which shows that the acceleration of a body has always the same direction as the force causing it.

---

- 1. What is force?
- 2. Is force a scalar or a vector quantity?
- 3. A body thrown upwards moves with an acceleration. Which force imparts it to the body? Which body is acting with this force? What is the direction of the force and acceleration?
- 4. Can we state, proceeding from the formula  $F = ma$  that the force  $F$  applied to a body "depends" on the mass  $m$  of the body and on its acceleration  $a$ ?
- 5. Can we say, on the basis of the formula  $a = F/m$ , that the acceleration  $a$  of a body depends on the force applied to it and on its mass?

---

## 4.7.

### What Do We Learn from Newton's Second Law?

UNIVERSAL LAW FOR ALL FORCES. We have obtained Newton's second law in the form  $\underline{\underline{F}} = m\underline{\underline{a}}$  from the experiment with a stretched spring. Is it valid only for elastic forces? It can be easily shown that this is not so. Let us consider Fig. 79 again. We have found that the accelerations  $\vec{a}$  and  $\vec{g}$  which would be imparted to the body by the elastic

force and the force of gravity are equal in magnitude and opposite in direction:  $\ddot{a} = -\ddot{g}$ . However, if the equality  $\ddot{a} = -\ddot{g}$  holds, the equality  $m\ddot{a} = -m\ddot{g}$  is also valid. As we know,  $m\ddot{a}$  is the elastic force, and hence  $m\ddot{g}$  represents the force of gravity. It is also equal to the product of the mass of the body and its acceleration. In the same way we can prove that Newton's second law is valid for forces of any origin.

**FORCE AND MOTION.** Newton's second law indicates that the *force applied to a body determines its acceleration, i.e. the change in the velocity rather than the velocity itself*. This means that the *force is the agency causing a change in motion (velocity) and is not the cause of motion. The direction of acceleration always coincides with that of force*. However, the direction of velocity, and hence the displacement of the body may differ from the direction of force.

If the force applied to a body coincides in direction with its velocity, the body moves in a straight line so that its velocity increases in magnitude. A body also moves rectilinearly when the vectors of force and velocity have opposite directions. But then the velocity decreases in magnitude. In both cases, if the magnitude and direction of a force are constant, the body is in a uniformly accelerated motion.

The force can also be directed at right angles to the velocity. In this case, the acceleration is also perpendicular to the velocity. It was shown above (see Sec. 3.2) that this is observed when a body is moving in a circle at a velocity of constant magnitude, viz. when the body has a centripetal acceleration (the velocity of the body changes only in direction). This is how the body was moving under the action of an elastic force in our experiment with a centrifugal machine. Thus, Newton's second law allows us to find out why and under which conditions bodies are in the motions studied in kinematics.

**A BODY IS ACTED UPON BY SEVERAL FORCES.** A body may be under the action of not one but several forces simultaneously. Experiments show that these forces "do not prevent" each other from imparting an acceleration to the body. The acceleration of the body turns out to be equal to the acceleration which a single force equal to the geometrical (vector) sum of all the forces would impart.

*The force equal to the geometrical sum of all the forces applied to a body is called the resultant force.*

In the formula  $\ddot{F} = m\ddot{a}$ ,  $\ddot{F}$  should be treated as the resultant of all the forces acting on a body. Let us consider an example. A boy on a rope swing (Fig. 83) experiences simultaneously the action of two forces: the force  $\ddot{F}_1$  directed downwards due to the action of the Earth and the force  $\ddot{F}_2$  caused by the rope and directed along it. Under the action of the two forces, the boy is moving in a circle around the pole to which the rope is tied. Therefore, the acceleration is directed towards the centre of the circle and not along  $\ddot{F}_1$  or  $\ddot{F}_2$ . It can be seen from the figure that the force  $\ddot{F}$  equal to the geometrical sum of  $\ddot{F}_1$  and  $\ddot{F}_2$  is also directed to the centre of the circle. Consequently, the boy moves in such a way as if only one force, i.e. the resultant  $\ddot{F}$  of the forces  $\ddot{F}_1$  and  $\ddot{F}_2$  is acting on him:

$$\ddot{F} = \ddot{F}_1 + \ddot{F}_2.$$

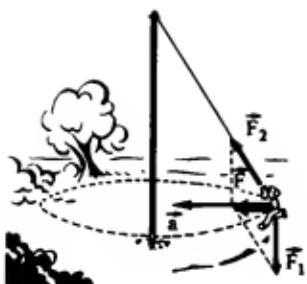


Fig. 83



Fig. 84

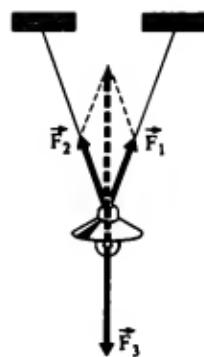


Fig. 85

**THE FORCES ARE PRESENT BUT ACCELERATION IS ZERO.** The vector sum of forces acting on a body can be equal to zero. Then, in accordance with Newton's second law, the body behaves as if no forces are acting on it. The acceleration of the body is equal to zero. This case was borne in mind when we spoke in Sec. 4.1 about the balance of influences (actions) of several forces on a given body. In the example with a small ball suspended from a string (see Fig. 67) the "balance" simply means that the sum of the forces applied to the ball is equal to zero (Fig. 84). Figure 85 illustrates the case when the resultant of not two but three forces ( $\vec{F}_1$ ,  $\vec{F}_2$  and  $\vec{F}_3$  acting on the lantern) is equal to zero.

**ANOTHER FORMULATION OF NEWTON'S FIRST LAW.** Using the concept of force, we can formulate *Newton's first law* in a different way.

There exist such reference systems relative to which a body in translatory motion conserves its velocity if the resultant of all the forces applied to the body is zero. Such reference systems were called inertial.

Newton's second law is also valid only for inertial reference systems.

**UNIT OF FORCE.** Formula  $F = ma$  which expresses Newton's law can be used for deriving the unit of force. Obviously, the force is equal to unity if under its action a body of unit mass acquires an acceleration equal to unity. Thus, the *unit of force in SI* is the force that imparts an acceleration of  $1 \text{ m/s}^2$  to a body of  $1 \text{ kg}$  mass. This force is called a newton (abbreviated as N)

$$1 \text{ N} = 1 \text{ kg} \cdot \text{m/s}^2.$$

?

1. What is the motion of a body acted upon by a force having a constant magnitude and direction?
2. What is the direction of the acceleration of a body due to a force acting on it?
3. Can a body acted upon by a single force move without an acceleration or be at rest?

4. Is it true that the velocity of a body is determined only by the force acting on it?
5. Can we state that a body always moves in the direction of a force applied to it?
6. Is the statement that the displacement of a body is determined only by the force acting on it correct?
7. Formulate Newton's first law using the concept of force.
8. A body is moving at a constant velocity. How will it move after the application of two forces equal in magnitude and opposite in direction?
9. A body of mass  $m$  is moving in a circle of radius  $r$  at a velocity  $v$  of constant magnitude. Is there a force acting on the body? What are the magnitude and direction of this force?

---

### Exercise 11

1. A body having a mass of 1.0 kg is falling to the Earth at a constant acceleration of  $9.8 \text{ m/s}^2$ . What is the magnitude of the force acting on it (force of gravity)?
2. A motorcar whose mass is 1000 kg is moving on a circular road of radius 100 m at a velocity of 20 m/s. What is the magnitude of the force acting on the car? What is its direction?
3. A car whose mass is 2160 kg starts moving with an acceleration which remains constant for 30 s. During this time, the car covers a distance of 500 m. What is the magnitude of the force applied during this time to the car?
4. Many years before Newton, the famous Italian artist and scientist Leonardo da Vinci put forth the following statement: "If a force displaces a body over a certain distance during a certain time, the same force would displace half of this body over the same distance during half this time". Is this statement true or false?

---

## 4.8.

### Measurement of Force

Force is one of the fundamental quantities in mechanics. This is so since knowing the force  $\bar{F}$  acting on a body of mass  $m$ , it is possible to calculate its acceleration  $\bar{a}$  by the formula

$$\bar{a} = \frac{\bar{F}}{m}.$$

As we know, the acceleration is the quantity required for solving the fundamental problem of mechanics. However, in order to find the value of the force it should be measured.

How can we measure the force acting on a body?

Let us recall how we measured the force of gravity with which the Earth acts on bodies near its surface (see *Junior Physics*, Sec. 31).

For this purpose, a body was suspended from a vertical spring. The spring was stretched to such an extent that the elastic force  $\bar{F}_{el}$  directed upwards along the axis of the spring balanced the force of gravity  $\bar{F}_g$ :

$$\bar{F}_{el} = -\bar{F}_g.$$

The elastic force  $\bar{F}_{el}$  with which the stretched spring acts on the body has been already known (see experiments described on pp. 84,85).

In these experiments the force of gravity acting on a body of mass  $m$  was found to be equal to  $mg$ . Therefore, the measurement of the force of gravity consisted in its balancing by a force which was known beforehand.

This method can be used for measuring any other force acting on any body. The force should be balanced by a known force applied to the body.

A spring is very convenient for measuring forces since it acts on *all bodies* with the same force when stretched (or contracted) to a certain extent. Besides, the same spring can be used for obtaining different forces if we stretch it to different extents.

In order to use a spring for measuring forces, we must first determine the magnitudes of elastic forces corresponding to its different elongations. In other words, we must find out how the elastic force depends on the elongation of the spring. For this purpose we could again use a centrifuge (place on it a spring with a body of known mass and measure the spring elongation at different speeds of rotation).

But since the magnitude of the force of gravity acting on a body is known, we can find in a simpler way the elastic forces corresponding to different elongations of a given spring.

We can suspend from a vertical spring bodies having different masses and measure each time the elongation of the spring with the help of a scale (Fig. 86). Indeed, we know that the magnitude of the force of gravity acting on a body of mass  $m$  is  $mg$ . When the body suspended to the spring is at rest, this force of gravity is balanced by the elastic force of the spring.

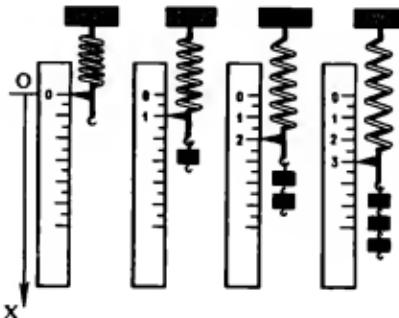


Fig. 86

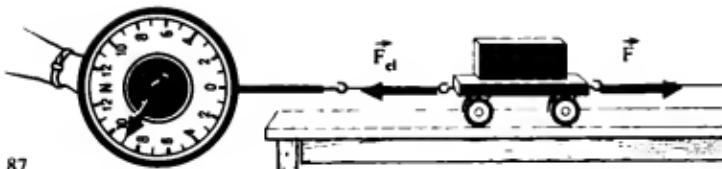


Fig. 87

Consequently, the magnitude of the elastic force of the spring is also  $mg$ .

In this way, we can establish the dependence of the elongation of the spring on the force of gravity acting on the body suspended from it. If against the divisions of the scale we write the figures corresponding to the value of the elastic force in newtons, the spring will be graduated. Such a graduated spring can be used for measuring any force. This device is called a *dynamometer*.

**HOW TO MEASURE A FORCE BY A DYNAMOMETER?** Suppose that a horizontal force  $\vec{F}$  which is to be measured is acting on a body (Fig. 87). We attach to this body the hook of a dynamometer with the horizontal axis. The dynamometer itself is at rest. Under the action of the force  $\vec{F}$ , the body acquires an acceleration and moves, entraining the hook of the dynamometer attached to it. The spring is elongated, thus giving rise to an elastic force directed against the force  $\vec{F}$ . When the elastic force  $\vec{F}_{el}$  and the force  $\vec{F}$  become equal in magnitude, the body stops and the pointer of the dynamometer indicates on the scale the value of the force  $\vec{F}$ .

It should be noted that the dynamometer with the body to which the force being measured is applied need not necessarily be at rest. Nothing will change if they move together uniformly in a straight line. We know that a uniform rectilinear motion also takes place when equal antiparallel forces are acting on a body. Figure 88 illustrates how the force with which the Earth (soil) is acting on a platform pulled by a tractor is measured. To ensure the accurate measurement, the tractor must move uniformly (at a constant velocity).

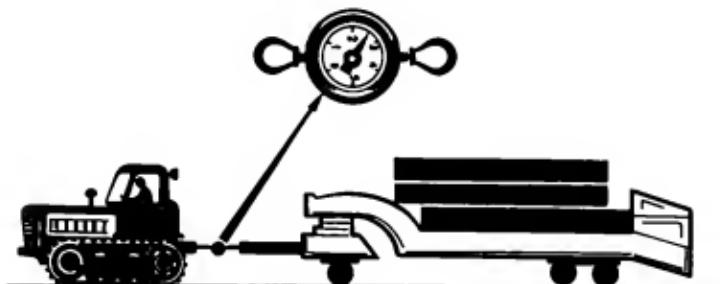


Fig. 88

## 4.9.

### Newton's Third Law

It was pointed out more than once that the actions of two bodies on each other are always mutual, i.e. the bodies interact. We can now say that each of the bodies acts on the other one with a certain force. For this reason, each body acquires an acceleration. It was shown in Sec. 4.5 that the ratio of the magnitudes of accelerations of two interacting bodies is equal to the inverse of their masses:

$$\frac{a_1}{a_2} = \frac{m_2}{m_1}.$$

Hence  $m_1 a_1 = m_2 a_2$ .

In the same section, it was pointed out that the accelerations imparted to the bodies during interaction have opposite directions. Hence we can write

$$m_1 \ddot{a}_1 = -m_2 \ddot{a}_2.$$

But  $m_1 \ddot{a}_1 = \ddot{F}_1$  and  $m_2 \ddot{a}_2 = \ddot{F}_2$ , where  $\ddot{F}_1$  and  $\ddot{F}_2$  are the forces acting on the first and second bodies. Consequently,

$$\ddot{F}_1 = -\ddot{F}_2.$$

This equality expresses *Newton's third law*:

Bodies act on each other with forces directed along the same straight line. These forces are equal in magnitude and opposite in direction

This law indicates that due to the "mutual" action of bodies on each other, the forces always emerge in pairs. If a force is acting on a body, there always exists another body on which the first body is acting with a force having the same magnitude but opposite direction. The accelerations imparted to the bodies by these forces also have opposite directions.

Newton's third law is valid for inertial reference systems.

The following experiment clarifies the meaning of Newton's third law.

Let us take two identical carts and fix an elastic steel plate to one of them. We bend this plate and tie it with a thread. Then we place the second cart in such a way that it is in close contact with the other end of the plate (Fig. 89). Let us cut the thread. The plate starts to straighten out and the two carts will be set in motion. This means that both of them have acquired an acceleration. Since the masses of the carts are equal, their accelerations are also equal in magnitude. Therefore, their velocities are also equal, as can be seen from the same displacements of the carts during the same time.

If we put a load on one of the carts (Fig. 90), we shall see that the displacements of the carts after releasing the plate will be different. This means that their accelerations are also different: the acceleration of the loaded cart will be lower.

This example, as well as any other, illustrates one more peculiarity of the two forces which, in accordance with Newton's third law, simultaneously emerge during the interaction of two bodies: these forces are always of the same origin. If, for example, the force exerted by one body on another is

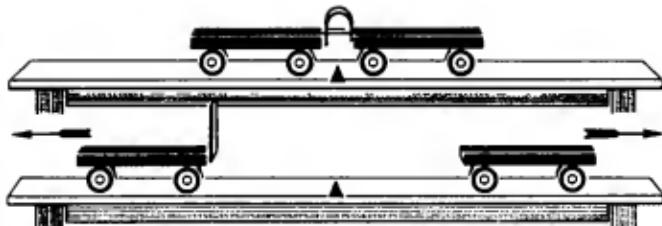


Fig. 89

elastic force, the second body "responds" to the first one by an elastic force as well.

It should be always borne in mind that the forces appearing during interaction of bodies are applied to different bodies and hence cannot balance each other. Only the forces applied to the same body can balance each other.

?

1. Formulate Newton's third law.
2. A formulation of the third law proposed by Newton himself is: "An action is always opposed by an equal reaction". Is there any physical difference between the action and reaction?
3. Do the forces of two interacting bodies balance each other?
4. Why is the damage to a motorcar colliding with a loaded lorry always more than the damage to the lorry?

#### EXAMPLE OF SOLVING A PROBLEM

The mass of each cart in the experiment shown in Fig. 90 is 200 g. The mass of the load on the right cart is 300 g. The elastic plate straightens out during 2 s, and the average elastic force  $\bar{F}_{av}$  of the plate is equal to 1 N. What are the displacements of the carts during this time? The mass of the plate and friction can be neglected.

*Solution.* We assume that during 2 s the plate acts with a constant force equal in magnitude to  $F_{av}$ .

Let the right cart move along the  $X$ -axis. The left cart experiences the action of the elastic force of the plate in the opposite direction. The projection of this force onto the  $X$ -axis is negative and equal in magnitude to  $F_{av}$ . According to Newton's third law, the right cart is acted upon by the force of the same magnitude but directed along the  $X$ -axis. Hence its projection onto the  $X$ -axis is positive. Then, in accordance with Newton's second law, we can write

$$-F_{av} = m_l a_{lx}, \quad F_{av} = m_r a_{rx},$$

where  $m_l$  and  $m_r$  are the masses of the left and right (loaded) carts, and  $a_{lx}$  and  $a_{rx}$  are the projections of the accelerations of the carts onto the  $X$ -axis.

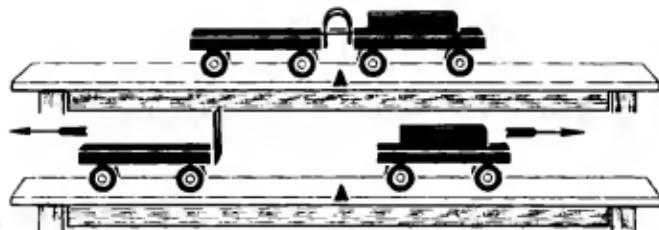


Fig. 90

Hence

$$a_{lx} = -F_{av}/m_l \quad \text{and} \quad a_{rx} = F_{av}/m_r.$$

The projections  $s_{lx}$  and  $s_{rx}$  of the displacements can be found from the formulas

$$s_{lx} = \frac{a_{lx}t^2}{2} = \frac{-F_{av}t^2}{2m_l}; \quad s_{rx} = \frac{a_{rx}t^2}{2} = \frac{F_{av}t^2}{2m_r}.$$

Substituting the values of the quantities from the condition of the problem, we get

$$s_{lx} = \frac{-1 \text{ N}(2s)^2}{2 \times 0.2 \text{ kg}} = -10 \text{ m}; \quad s_{rx} = \frac{1 \text{ N}(2s)^2}{2 \times 0.5 \text{ kg}} = 4 \text{ m}.$$

### Exercise 12

1. Two men pull a rope in opposite directions with a force of 50 N each. Will the rope break if it can withstand a tension of 80 N?
2. Two boys weighing 40 and 50 kg stand facing each other on rollerskates. The first boy is pushing away from the second boy with the force of 10 N. Find the accelerations of the boys.

## Summary. The Importance of Newton's Laws

Our experience and observations show that the cause of a change in motion of bodies, i.e. the cause of a change in their velocity are the actions of other bodies on them. Without this action, the motion of a body cannot change, i.e. no acceleration can appear. The action of one body on another is quantitatively expressed by the quantity called *force*.

The action of one body on another is not unilateral. Bodies act on each other, i.e. they are *interacting*. The acceleration of a body due to interaction depends on a peculiar property of the body, viz. its inertia which is expressed by the quantity called *mass*.

These experimental results form the basis of three laws of motion (dynamics), discovered by Newton at the end of the 17th century. These laws are strikingly brief and simple if motions are considered in appropriate reference systems, viz. inertial systems.

*Newton's first law* states that such systems do exist and enables us to find them.

There exist such reference systems relative to which the velocity of a body in translational motion remains unchanged if the sum of the forces acting on the body is equal to zero.

*Newton's second law* establishes the relation between the force and acceleration caused by it.

Regardless of its nature, the force acting on a body is equal to the product of the mass of the body and the acceleration imparted by this force:

$$\vec{F} = m\vec{a}.$$

*Newton's third law* shows that the action of a body on another is of mutual nature.

Bodies act on each other with forces of the same origin, which are equal in magnitude and opposite in direction:

$$\vec{F}_1 = -\vec{F}_2.$$

The laws of motion are expressed by two simple (at first sight) formulas. However, they contain an extraordinary rich information. Different kinds of motion occur around us: water flows in rivers, waterfalls come down, winds and hurricanes blow over the Earth, motorcars run on the roads, ships sail in the seas, airplanes fly in air, galaxies, stars, planets, and artificial satellites are in motion in space. These motions as well as bodies are different. The forces acting on the bodies are also different. However, Newton's laws are equally valid for all these motions, bodies and forces. These laws are expressed analytically in the above formulas which seem to be very simple.

Newton's mechanics was the first complete theory in the history of physics (and science in general) which correctly described a large class of phenomena, viz. the motion of bodies. Not without reason, one of Newton's contemporaries expressed his admiration at this theory in the following verses:

Nature and Nature's Laws  
Lay hid in night;  
God said, *Let Newton be!*  
And all was light.

Newton's laws permit, in principle, to solve any problem in mechanics. If the forces applied to a body are known, its acceleration can be found at any point of the trajectory at any instant of time.

Thus, the "chain" mentioned at the end of Chapter 3 has been completed: from known forces and mass of the body, its acceleration is determined. Then the velocity of the body and its displacement over any interval of time are calculated. Finally, the coordinates of the body at any instant of time are

found. For this we must know the "initial conditions", viz. the initial position and initial velocity of the body.

For example, the scientists who control the flight of a spacecraft obviously must know in advance the position of the spacecraft at any instant of time. They can find it by using such a "chain". They know the initial position of the spacecraft on the launching pad and its initial velocity. They also know the forces acting on the spacecraft at any point of the trajectory. Using these data, they solve the problem of mechanics for a given flight. But since the forces acting on the spacecraft are varying all the time, the calculations become so complicated that they can be made only with the aid of computers.

We repeatedly stated that the basic problem of mechanics is to determine the position of a moving body at any instant of time. However, this does not mean that the laws of motion are used just for calculating the position of a body. In actual practice, such quantities as the velocity of a body, its acceleration, forces acting on it, etc. have to be found. Of course, Newton's laws allow us to solve such simpler problems as well.

# 5

## FORCES IN NATURE

### ARE THERE MANY TYPES OF FORCE IN NATURE?

It was pointed out earlier that the cause of a change in motion, i.e. the cause of acceleration of bodies is force. On the other hand, forces emerge when bodies interact. Which kinds of interactions exist in nature? Are these kinds numerous?

At first sight, there is a large number of force types the bodies act on each other, and hence, there are many types of interactions. We can impart an acceleration to a body by pushing or pulling it by the hand. Any body falling to the Earth moves with an acceleration. A ship whose sails are blown up by the wind starts moving with an acceleration. Stretching and releasing the bow string, we impart an acceleration to the arrow. In all these cases some forces are acting, which seem to be quite different. Naturally, we can mention many other forces also. Everybody knows about electric and magnetic forces, forces of earthquakes, tides, etc.

Are there indeed so many different types of forces in nature? It turns out that this is not so.

While considering mechanical motion of bodies, we have to deal only with three types of forces: *elastic force*, *force of friction* and *gravitational force*. All the forces mentioned above, which seem so different, are reduced to one of these types. However, these three types of forces are the manifestation of only two main forces in nature, which are indeed of different origin. These are *electromagnetic forces* and *gravitational forces*.

Let us first consider electromagnetic forces.

From the introductory course of physics it is known that there is a force which is acting between electrified bodies and called *electric force*.

Electric forces can be either attractive or repulsive. Charges of two types exist in nature. They are conventionally called positive and negative charges. Two bodies with like charges repel and with unlike charges attract each other.

Electric charges possess a very interesting property. When they are moving relative to each other, one more force is acting between them in addition to an electric force. This is a *magnetic force*.

These two forces—electric and magnetic—are related to each other so closely that they cannot be separated: they are acting *simultaneously*. Since we deal as a rule with moving charges, the forces acting among them can be called neither electric nor magnetic. They are called *electromagnetic forces*.

What is the origin of an “electric charge” which a body may have or have not?

Any body consists of molecules and atoms. Atoms, in turn, (although they are very small—of the order of  $10^{-8}$  cm) are composed of still smaller particles, viz. atomic nuclei and electrons. These particles, nuclei and electrons, possess electric charges. A nucleus has a positive charge while electrons are charged negatively.

Under normal conditions, an atom contains such a number of electrons that their total negative charge is equal to the positive charge of the nucleus so that the atom on the whole as if has no charge. It is said to be electrically neutral. Hence the bodies composed of such neutral atoms are also electrically neutral. Electric forces are practically absent when such bodies interact.

On the other hand, the neighbouring atoms constituting a solid (or liquid) are arranged so closely that the forces of interaction between the charges of these atoms are considerable.

Forces of interaction of atoms depend on their separations. This dependence is complex and is not exactly known so far. However, the forces of interaction of atoms are found to change the direction upon a change in the separation between them. If it is very small, atoms repel each other. If, however, the separation of the atoms has increased, they start to attract each other. At a certain atomic separation, the forces of interaction between the atoms become equal to zero (vanish). Naturally, atoms are arranged relative to each other just at such separations in solids and liquids. It should be noted that these distances are very small. They are of the order of atomic dimensions.

## 5.1. Elastic Forces

When a body is stretched, the atomic separation somewhat increases, and attractive forces emerge between the atoms. These forces impart acceleration to the atoms and make them come closer up to the previous distance.

If, on the contrary, a body is compressed and thus its atoms are brought closer, repulsive forces emerge, which make the atoms move apart and occupy the previous positions.

Thus, if a body is stretched or compressed, the forces of electrical origin emerge in it, which restore the previous dimensions of the body.

Such restoring forces also appear when bodies are bent (Fig. 91) or twisted (Fig. 92) because in these cases too the mutual arrangement of atoms changes.

*Extension, compression, bend and torsion are called deformations of bodies.* Experiments show that in any deformation (if it is not too large in comparison with the dimensions of the body itself) a force emerges, which returns the body to the initial state prior to the deformation. It is this force that is called *elastic force*.

In Secs. 4.7 and 4.9 we considered elastic forces appearing upon deformation of a spring. We can now say that an elastic force emerges dur-

ing deformation of any body and not only a spring (each body may play the role of a spring!).

Since an elastic force returns the body to the initial state, it is directed oppositely to the displacement of particles of the body during deformation. If, for example, a rod with one fixed end (Fig. 93a) is stretched so that its particles are displaced to the right of the fixed end (Fig. 93b), the elastic force is directed to the left. If the rod is compressed as shown in Fig. 93c, its particles are displaced to the left and the elastic force is directed to the right.

*Elastic force is the force appearing during deformation of a body and having the direction opposite to the displacement of particles of the body*

Hence force, we shall be considering elastic forces emerging only during extension or compression.

The change in the length of the body in Fig. 93c (its elongation) is denoted by  $x$ . Figures 93b and c show that  $x$  is also the projection onto the  $X$ -axis of the displacement vector of the free end of the rod upon extension or compression. This projection is positive for the extension of the rod and negative for its compression.

**HOOKE'S LAW.** Experiments similar to that described in 4.9 (see Fig. 86) were carried out not only with springs but also with rigid rods. These experiments have led to the relation between elastic force and deformation causing it. It turned out that for sufficiently small elongations (small in comparison with the length of the rod), the magnitude of the elastic force vector is proportional to the magnitude of the displacement vector for the free end of the rod. As we know (see Figs. 93b and c), the projections of these vectors onto the  $X$ -axis have opposite signs. Hence the analytical form of this dependence is

$$(F_{\text{el}})_x = -kx. \quad (5.1.1)$$

Here  $k$  is the proportionality factor called the *rigidity* of the body (or spring). The rigidity depends on the dimensions of a body (spring) and the material of which it is made. The unit of rigidity in SI is newton per metre (N/m).

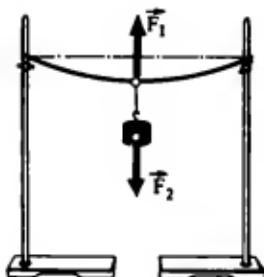


Fig. 91

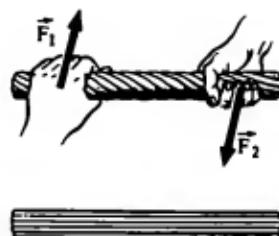


Fig. 92

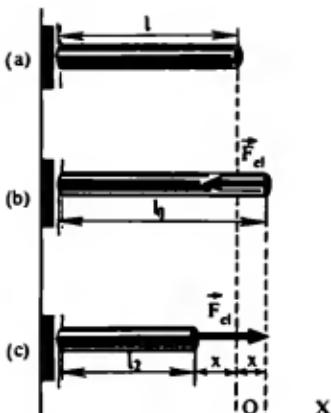


Fig. 93

Formula (5.1.1.) expresses *Hooke's law*: Elastic force due to deformation of a body (spring) is proportional to the elongation of the body and is directed oppositely to the displacement of particles of the body under deformation.

It follows from what is said above that the elastic force depends on coordinates of some parts of a body relative to other parts.

---

9

1. List the types of interactions in nature. Which of them cause an elastic force?
2. Which of the forces cited at the beginning of this chapter are elastic?
3. Under which conditions do elastic forces emerge?
4. Formulate Hooke's law.

---

### Exercise 13

1. A load of mass 0.1 kg is suspended from a vertical spring whose upper end is fixed. After the vibrations of the load have damped, it turns out that the spring is elongated by 2 cm. What is the rigidity of the spring?
2. Two identical carts having the mass of 100 g each are connected through a compressed spring whose length (in the compressed state) is 6 cm. The rigidity of the spring is 30 N/m. After the spring had expanded, the carts moved apart at an acceleration of  $6 \text{ m/s}^2$ . Find the length of the undeformed spring.

---

## 5.2.

### Motion Is the Cause of Deformation

How does the deformation of a body emerge? Let us take two carts with balls of soft rubber fixed in front of them (Fig. 94). We push the carts towards each other so that they collide. When the balls touch each other, they both change their shape, i.e. get deformed. At the same time, the velocities of the carts to which the balls are fixed are gradually decreasing. Ultimately, the carts stop for a moment and then start to move backwards, reversing the directions of their accelerations. Obviously, the cause of acceleration is the elastic force due to deformation of the balls. This experiment shows that the deformation occurred because the balls, after having been brought in contact, continued to move for a certain time in the previous direction until the elastic force due to deformation stopped them. After this, the deformed balls restoring their shape made the carts move in the opposite directions. However, as soon as the balls had restored their shape, the elastic force vanished. Hence, we can state that *the cause of the deformation of the ball was the motion of one its part relative to the other and as a consequence of deformation, the elastic force appeared*.

If we now replace rubber balls by steel ones and repeat the experiment, the result is just the same. The carts collide, stop for a moment and then start to move in the opposite directions. However, in this experiment we do not see the change in the shape of the balls (their deformation). But this does not mean that there is no deformation since the carts with steel balls behave just in the same way as the carts with rubber balls. As a matter of fact, the deformations of steel balls are very small and cannot be detected without special devices.

Not only deformations but also the motions causing them are often imperceptible. For example, when we are looking on a book (or some other load) lying on the table, we naturally cannot notice that both the load and the table are slightly deformed. But it is just the deformation of the table,

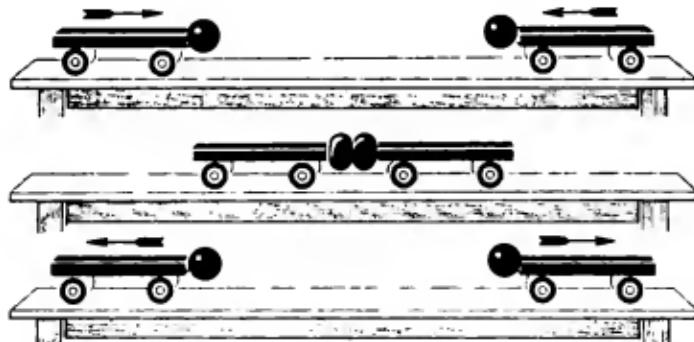


Fig. 94



Fig. 95

completely imperceptible by the naked eye, leads to the emergence of the elastic force directed upwards and balancing the attraction of the load to the Earth. For this reason, the load remains at rest. When we put a load on a table, it starts to move downwards due to the attraction to the Earth, like any falling body. The moving load displaces the particles comprising the part of the table which is in contact with the load. The table is deformed, which gives rise to the elastic force just equal to the force of attraction of the load to the Earth but directed upwards.

If we put the load on a soft rubber, we can see by a naked eye both the displacement and the final deformation of the rubber (Fig. 95).

The same line of reasoning applies to the action of a suspender (Figs. 96a and b).

In many cases, the deformations causing an elastic force are quite significant. The elongation of a spiral or a rubber cord can be easily observed. Using rapid photography, we can observe the deformation of a football under the impact of a player. Coloured plate IIa shows the shape acquired by the round ball at the moment of impact. Tennis ball also loses its spherical shape under the blow of a racket.

Elastic force exerted on a body by a support or suspender is often called the *reaction of support* or the *reaction of suspender* (*suspender tension*).

The above examples show that elastic force emerges during the contact of two interacting bodies. Of course, both bodies are always deformed.

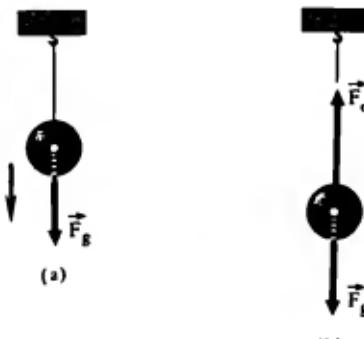


Fig. 96



Fig. 97



Fig. 98

An important property of elastic force is that it is directed *normally to* the surface of contact of interacting bodies. If such bodies as compressed or stretched springs are taking part in an interaction, the elastic force is directed along their axes.

?

1. Under which conditions do deformations of bodies appear?
2. Figure 97 shows an archer. What is the direction of an elastic force imparting an acceleration to the arrow?
3. A load is at rest on an inclined plane (Fig. 98). Is an elastic force acting on the load? Which deformed body is responsible for this force?
4. What is called the reaction of support?

#### Hometask

Explain the appearance of elastic force  $\vec{F}_{el}$  in the experiment illustrated in Figs. 96a and b.

## 5.3.

### Force of Universal Gravitation

Newton discovered the laws of motion of bodies. According to these laws, accelerated motion is possible only under the action of a force. Since falling bodies move with an acceleration, they should experience the action of a downward force directed to the Earth. Is it only the Earth that has the property to attract bodies near its surface? In 1667, Newton put forward the hypothesis that forces of mutual attraction are in general acting among all bodies. He called these forces the *forces of universal gravitation*.

Then why don't we notice mutual attraction between bodies surrounding us? Perhaps, this is due to the fact that attractive forces are too weak?

Newton managed to show that the force of attraction between two bodies depends on their masses. It turned out that its value becomes significant only when interacting bodies (or at least one of them) have an appreciable mass.

**THE ROLE OF MASSES OF INTERACTING BODIES.** Free fall acceleration has a remarkable property consisting in that it is the same for all bodies (of any mass) at a given point. How can this peculiar property be explained?

The only explanation of the fact that the acceleration due to gravity is independent of the mass of a body is that *the force  $F$  itself, with which the Earth attracts the body, is proportional to its mass.*

Indeed, in this case, say, a double increase in the mass  $m$  will double the magnitude of the force  $F$ , while the acceleration equal to the ratio  $F/m$  remains unchanged. Newton draw a single correct conclusion: the force of universal gravitation is proportional to the mass of the body on which it is acting.

However, there is a mutual attraction between bodies, the forces of interaction being of the same origin. Consequently, the force with which the body attracts the Earth is also proportional to the Earth's mass. According to Newton's third law, these forces are equal in magnitude. Hence, if one of them is proportional to the mass of the Earth, the second force equal to the first one (viz. the force with which the Earth attracts the body) is also proportional to the mass of the Earth. Therefore, the force of mutual attraction is proportional to the masses of the two interacting bodies. This means that *it is proportional to the product of masses of the bodies.*

What else determines the force of mutual attraction of two bodies?

**THE ROLE OF THE DISTANCE BETWEEN THE BODIES.** Newton supposed that the force of mutual attraction between two bodies must depend on the distance between them. It is well known from experiments that near the Earth's surface, the free-fall acceleration is  $9.8 \text{ m/s}^2$  and it is the same for all bodies falling from the height of 1, 10 or 100 m. We cannot conclude from this, however, that the acceleration is independent of the distance from the Earth. Newton believed that the distance should be measured not from the Earth's surface but from its centre. The radius of the Earth is 6400 km. Hence it is clear that several tens of metres above the Earth cannot introduce any noticeable change in the free-fall acceleration.

In order to determine the effect of the separation of bodies on their mutual attraction, we must know the acceleration of bodies at large distances from the Earth's surface.

It is obviously difficult to measure the free-fall acceleration of bodies at an altitude of several thousand kilometres above the Earth' surface. It would be more convenient to measure the centripetal acceleration of a body moving in a circle around the Earth under the action of attraction by the Earth. (Recall that the same technique was used by us while studying the elastic force.) We measured the centripetal acceleration of the cylinder moving in a circle under the action of this force (see Sec. 4.6).

Nature itself came to help physicists studying the force of universal gravitation and made it possible to determine the acceleration of a body

moving in a circle around the Earth. This body is the natural satellite of the Earth – the Moon. Indeed, if Newton's hypothesis is correct, it should be assumed that it is the force of attraction to the Earth that imparts to the Moon a centripetal acceleration in its orbiting the Earth. If the gravitational force between the Moon and the Earth were independent of their separation, the centripetal acceleration of the Moon would be the same as the acceleration due to gravity for the bodies near the Earth's surface. In fact, the centripetal acceleration of the orbital motion of the Moon is known to be  $0.0027 \text{ m/s}^2$  (see Problem 5 of Exercise 8). This is about  $1/3600$  of the free-fall acceleration for bodies on the Earth. At the same time, the centre-to-centre distance for the Earth and the Moon is known to be about 384 000 km. It is 60 times as large as the Earth's radius, i.e. the distance from its centre to the surface.

Thus, the 60-fold increase in the distance between bodies attracting each other decreases the acceleration by a factor of  $60^2$ .

Hence we may conclude that the acceleration due to the force of universal gravitation, and consequently this force itself, is inversely proportional to the squared separation of the interacting bodies. This was just the conclusion drawn by Newton.

**THE LAW OF UNIVERSAL GRAVITATION.** We can thus write that two bodies having masses  $M$  and  $m$  attract each other with a force  $\vec{F}$  whose magnitude is expressed by the formula

$$\underline{F = G \frac{Mm}{r^2}}, \quad (5.3.1)$$

where  $r$  is the separation of the bodies and  $G$  is the proportionality factor, which is the same for all bodies in nature. This factor is called the *constant of universal gravitation, or gravitational constant*.

Formula (5.3.1) expresses the *law of universal gravitation* discovered by Newton.

All bodies attract one another with a force whose magnitude is directly proportional to the product of their masses and inversely proportional to their squared separation.

The force of universal gravitation makes planets move around the Sun and artificial satellites, around the Earth.

What should be meant under the separation of interacting bodies?

It turned out that formula (5.3.1) which expresses the law of universal gravitation is valid when the separation of the bodies is so large in comparison with their dimensions that the bodies can be treated as material points. This force is directed along the straight line connecting the material points. While calculating the gravitational force, the Moon and the Earth, the planets and the Sun can be treated as material points.

If the bodies have spherical shape, they attract one another as material points located at their centres even when their dimensions are comparable with their separation. In this case,  $r$  is the centre-to-centre distance of the spheres and the force is acting along the line connecting the two centres.

Formula (5.3.1) can also be used for calculating the force of attraction

between a large sphere and a body having a small size and arbitrary shape, which is near the surface of the sphere. Then we can neglect the dimensions of the body in comparison with the radius of the sphere. We proceed just in this way while considering attraction of different bodies and the globe.

Gravitational force is another example of the force which depends on the mutual arrangement of interacting bodies, i.e. on their coordinates (the gravitational force depends on the separation  $r$  between the bodies).

## 5.4. Gravitational Constant

The formula expressing the law of universal gravitation contains the coefficient  $G$ , viz. the constant of universal gravitation (gravitational constant). What is the meaning of this quantity?

The coefficient  $G$  has a simple meaning. If the masses  $M$  and  $m$  of two interacting bodies are equal to unity ( $M = m = 1$  kg) and their separation is also equal to unity ( $r = 1$  m), it follows from formula (5.3.1) that the force  $F$  is equal in magnitude to the gravitational constant  $G$ .

What are the dimensions of constant  $G$ ? The formula expressing the law of universal gravitation leads to the following expression:

$$G = \frac{Fr^2}{Mm}.$$

If the force is given in newtons (N), the distance in metres (m) and the mass in kilograms (kg), the quantity on the right-hand side of this equation has the dimensions  $N \cdot m^2/kg^2$ . We know that in any formula (if it is correct) the quantities on different sides of equation should be expressed in the same units (for example, 5 m cannot be equal to 5 kg). Hence it follows that constant  $G$  must have the dimensions of  $N \cdot m^2/kg^2$ .

The numerical value of the gravitational constant can be determined only from an experiment in which the force  $F$  acting on one of the bodies of known masses  $m_1$  and  $m_2$  arranged at a known distance  $r$  should be determined in one way or another.

Such experiments have been carried out more than once. In one of them a glass ball filled with mercury was suspended from a long thread fixed to a pan of sensitive scales (beam balance) (Fig. 99). The other pan was loaded with weights for balance. After the scales had been thoroughly balanced, a lead sphere of a large mass (about 6000 kg) was arranged under the sphere with mercury as close to it as possible. The equilibrium of scales was violated due to attraction of the mercury sphere to the lead one. In order to balance the scales, an additional load should be placed on the pan with weights. The force of attraction between this additional weight and the Earth is obviously equal to the force of attraction between the mercury sphere and the lead one, i.e.

$$F = G \frac{m_1 m_m}{r^2}.$$

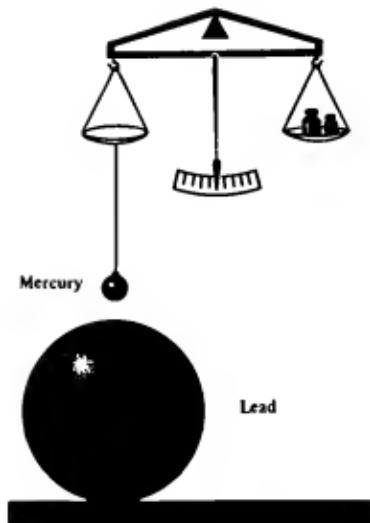


Fig. 99

Here  $m_l$  is the mass of the lead sphere,  $m_m$  is the mass of the mercury sphere and  $r$  is their centre-to-centre distance. Hence we can easily calculate the value of  $G$ :

$$G = \frac{Fr^2}{m_l m_m}.$$

This and many other experiments have yielded the following value of  $G$ :

$$G = 6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2.$$

This is a very small quantity. It is because of its smallness that we do not notice the attraction among bodies surrounding us. Two balls having a mass of one ton each and located at a distance of 1 m attract one another with a force equal only to  $6.67 \times 10^{-5}$  N.

?

1. What will be the change in the force of attraction between two spheres if (a) one of them is replaced by a sphere whose mass is twice larger? (b) the mass of the second sphere is also doubled?
2. What will be the change in the force of attraction between two spheres if their centre-to-centre distance is doubled?
3. Bodies on the Earth's surface attract one another. Why don't we notice this?
4. Which force makes the Earth and other planets move around the Sun?

---

### Exercise 14

1. The mass of the mercury sphere described in this section is 5 kg, its radius is 4.5 cm, the mass of the lead sphere is 6 t and its radius is 0.5 m. Which load should be added on the right pan of the balance to compensate the attraction between the lead and mercury spheres?
2. Two ships 50 000 t each are on harbour at a distance of 1 km from each other. What is the force of their attraction?
3. Calculate the force of attraction between the Moon and the Earth. The mass of the Moon  $m_M \approx 7 \times 10^{22}$  kg, the mass of the Earth  $m_E \approx 6 \times 10^{24}$  kg, and their separation should be taken at  $3.84 \times 10^8$  m.
4. A cosmonaut has landed on the Moon. He is being attracted both by the Moon and by the Earth. What is the ratio of the forces of attraction to the Moon and the Earth? The radius of the Moon is 1730 km.

---

## 5.5.

### Force of Gravity

**FORCE OF GRAVITY.** One of manifestations of the force of universal gravitation is the force of gravity, viz. the force of attraction to the Earth. We denote the Earth's mass by  $M$ , its radius by  $R$  and the mass of a body by  $m$ . Then the force acting on the body near the Earth's surface is, in accordance with the law of universal gravitation

$$F = G \frac{Mm}{R^2}. \quad (5.5.1)$$

This is just the force of gravity. It is directed to the centre of the Earth. If only this force is acting on a body (while all other forces are mutually balanced) the body falls freely. The free-fall acceleration can be found with the help of Newton's second law:

$$g = \frac{F}{m} = G \frac{Mm}{R^2 m} = G \frac{M}{R^2}. \quad (5.5.2)$$

Thus the *free-fall acceleration  $g$  does not depend on the mass  $m$  of the body* and consequently it is the same for all bodies. This is a remarkable property of the force of universal gravitation, and hence of the force of gravity, which was discovered experimentally by Galilei. It is remarkable since, according to Newton's second law, the acceleration should be inversely proportional to the mass. However, the gravitational force itself is proportional to the mass of a body on which it acts. This is the reason behind the fact that the free-fall acceleration is the same for all bodies.

We can now write the following equation for the force of gravity:

$$\bar{F}_g = m\bar{g},$$

which was derived earlier (see Sec. 4.7).

**FREE-FALL ACCELERATIONS ARE DIFFERENT AT DIFFERENT LATITUDES.** Strictly speaking, formula (5.5.2), as well as Newton's second law, is valid when a free fall is considered in an inertial reference system. On the surface of the Earth, reference systems fixed to the poles of the Earth, which do not take part in its diurnal rotation, may serve as inertial systems of reference. All other points of the Earth's surface are moving in a circle with centripetal accelerations. Hence, reference systems fixed to these points are not inertial and Newton's second law is inapplicable to them.

Rotation of the Earth leads to the fact that the free-fall accelerations measured relative to any body fixed on the Earth's surface are different at different latitudes.

Another important cause of the difference in free-fall accelerations at different points on the Earth's surface is due to the fact that the globe is slightly flattened at the poles.

Experiments show that the free-fall acceleration measured relative to the Earth's surface at the poles is about  $9.83 \text{ m/s}^2$ , on the equator it is  $9.78 \text{ m/s}^2$ , while at the latitude of  $45^\circ$  it is  $9.81 \text{ m/s}^2$ .

These figures show that the values of free-fall acceleration in different regions of the globe differ only slightly from each other and from the value calculated from the formula

$$g = G \frac{M}{R^2} \approx 9.83 \text{ m/s}^2.$$

Hence, in rough calculations the facts that reference systems connected with the Earth's surface are noninertial and that the shape of the globe differs from spherical are neglected. The free-fall acceleration is assumed to be the same and is calculated by formula (5.5.2).

In some regions of the globe, the free-fall acceleration differs from the above value for some other reason. The anomalies are observed in the regions where the Earth's interiors contain deposits of minerals whose density is higher or lower than the average density of the Earth. The value of  $g$  is larger where denser minerals are deposited. This gives a tool for geologists to find the deposits of commercial minerals by measuring the value of  $g$ .

Finally, the force of gravity, and hence the free-fall acceleration change as a body is moving away from the surface of the Earth. If a body is at a height  $h$  above the Earth's surface, the magnitude of the free-fall acceleration  $g$  should be written in the form

$$g = G \frac{M}{(R + h)^2}.$$

For example, if a body is at a height of 300 km, the acceleration due to gravity is  $1 \text{ m/s}^2$  smaller than at the surface. This formula shows that the

force of gravity can be assumed constant, independently of the position of a body, at altitudes above the Earth not only of dozens or hundreds of metres but also of several kilometres. It is only for this reason that free fall near the Earth can be assumed to be a *uniformly accelerated motion*.

**MEASUREMENT OF THE MASS OF A BODY BY WEIGHING.** It was shown in Chapter 4 that the mass of a body can be determined by measuring the ratio of accelerations acquired during the interaction of the given body with the one taken as the standard of mass. Obviously, this method is very inconvenient and normally never employed in actual practice. Let us now consider another, more convenient method for measuring the mass. This method is called *weighing*. It is based on the fact that the force of gravity acting on a body and the mass of this body are proportional to each other:

$$F_g = mg.$$

The force of gravity can be measured by a dynamometer (spring balance). Having measured the gravity force  $F_g$  and knowing the value of  $g$  at the place where weighing is performed, we find the mass of the body from the formula

$$m = \frac{F_g}{g}.$$

It is still more convenient to determine the mass of a body by weighing it on a *beam balance* when the mass of a body (on one pan) is compared with the mass of weights (on the other pan). When the balance is in equilibrium, we can state that the same force of gravity is acting on the body and on the weights. This means that the mass of the body is also equal to the mass of the weights. Since it is the mass which is indicated on a weight, we can determine the mass of the body by summing up the figures indicated on the weights.

Beam balance is a very sensitive instrument. The smallest mass that can be measured with the help of a modern sensitive balance amounts to several millionths of a kilogram.

---

?

1. Give the definition of the force of gravity.
2. Free-fall acceleration of bodies is independent of their mass. What about the force of gravity?
3. Is the force of gravity the same at different points on the globe?
4. Does the rotation of the Earth around its axis affect the force of gravity?
5. Does the force of gravity acting on a body change when it moves away from the Earth's surface?
6. What is the direction of the gravity force acting on any body?

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## EXAMPLE OF SOLVING A PROBLEM

Calculate the mass of the Earth if the free-fall acceleration near its surface is known to be  $9.8 \text{ m/s}^2$ . Assume that the radius of the globe is 6370 km.

*Solution.* Naturally, the mass of the Earth cannot be measured with the help of a beam balance. However, it can be calculated from the formula for the free-fall acceleration:

$$g = G \frac{M}{R^2},$$

where  $G = 6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2$  is the gravitational constant.

Hence, the mass of the Earth

$$M = \frac{gR^2}{G}.$$

Substituting the values of  $g$ ,  $R$  and  $G$ , we obtain

$$M = \frac{9.8 \text{ m/s}^2 (6.37 \cdot 10^6 \text{ m})^2}{6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2} \simeq 6.0 \times 10^{24} \text{ kg}.$$

The mass of the Earth is about six million billion billion kilograms!

---

### Exercise 15

1. What is the mass of a body if the force of gravity acting on it is 49 N? The body is near the Earth's surface.
2. At what altitude above the Earth does the force of gravity acting on a body decrease by half?
3. Find the force of attraction acting on one-kilogram body near the Moon's surface. What is the ratio between this force and the force of gravity acting on this body at the Earth's surface?
4. Calculate the free-fall acceleration near the Mars surface, assuming that the mass of the Mars is  $6.0 \times 10^{23} \text{ kg}$  and its radius is 3300 km.

---

## 5.6.

### Friction. Static Friction

We have already mentioned one of the manifestations of electric interaction between bodies, viz. elastic force. *Friction* mentioned by us more than once is another manifestation of electric interaction. This force cannot be forgotten since it accompanies any motion of bodies. It is due to this force that motion of a body ultimately ceases unless any other force is acting on the body, like elastic force or force of gravity.

It should be recalled (see *Junior Physics*, Sec. 34) that friction emerges when bodies are in direct contact and is always directed along the contact surface unlike elastic force which is always normal to this surface.

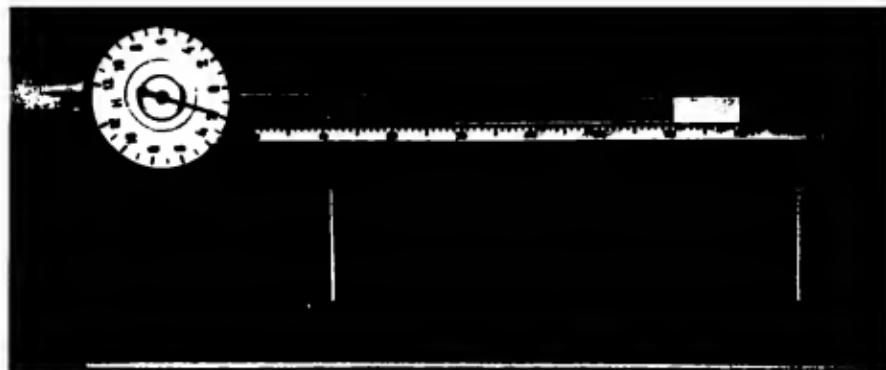


Fig. 100

Let us illustrate the emergence of friction by the following experiment (Fig. 100) A dynamometer is attached to a body resting on a support. The dynamometer string is stretched by the hand. Figure 101 schematically shows the forces acting on the body: the force  $\vec{F}$  parallel to the surface of contact between the body and the table. This force is measured by the dynamometer. Besides, the body is acted upon by the force of gravity  $\vec{F}_g$  and the normal reaction of support  $\vec{N}$ , balancing the force of gravity. This force is due to deformation of the table and is directed *normally* to the contact surface between the body and the table. If the load is insufficient, the body remains at rest. This means that another force  $\vec{F}_f$  is acting on the body in addition to the force  $\vec{F}$ . This force is equal in magnitude to  $\vec{F}$  but has the opposite direction

$$\vec{F}_f = -\vec{F}$$

This is just the friction which is called *static friction*.

Let us increase the load by attaching to it a large weight. The dynamometer indicates an increase in force  $\vec{F}$ . However, the body, as before, remains at rest. This means that the force of static friction has increased together with  $\vec{F}$  so that, as before, these forces are equal in magnitude and have opposite direction. This is the main peculiarity of static friction.

*Static friction is always equal in magnitude and directed oppositely to the force applied to the body along the surface of its contact with another body.*

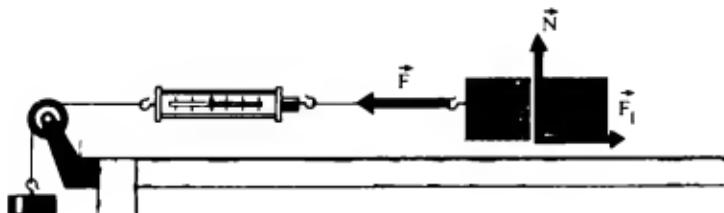


Fig. 101

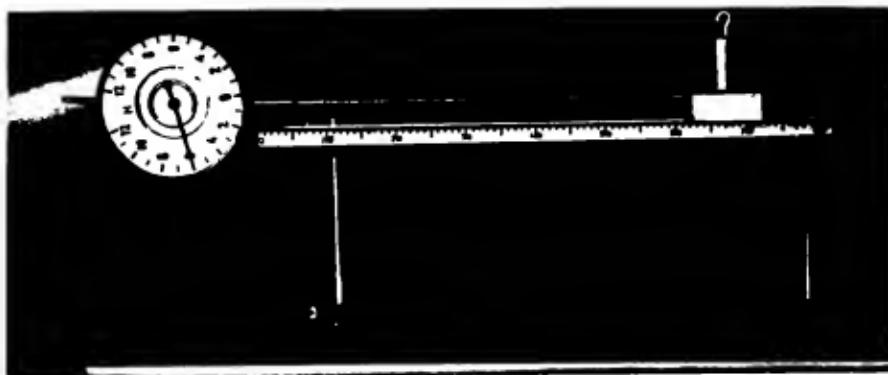


Fig. 102

Finally, at a certain value of the mass of the load, the body starts to slide. Consequently, there exists a certain maximum static friction  $F_{f\max}$ . Only when the force  $F$  parallel to the surface becomes if only slightly larger than static friction, the body acquires an acceleration

Static friction is just the force that prevents us from shifting a heavy object a cupboard, a table, a drawer, etc.

Why is it important that the body should be heavy? We are not moving it upwards, against the force of gravity are we? The answer to this question is given by an experiment

Let us place a load on a body in order to press it against the table (Figs. 102 and 103) (instead we could press it by the hand, spring, etc.). Thus we increase the force directed normally to the surface of contact between the body and the table (Force applied to the surface of support in contact with the body is called the *force of pressure*). If we now measure again the maximum static friction, i.e. the force required to make the body slide, it turns out to increase in the same proportion as the force of pressure.

*Maximum static friction is proportional to the force of pressure*

In accordance with Newton's third law, the force of pressure of a body on a support is equal in magnitude to the normal reaction. Hence, we can write

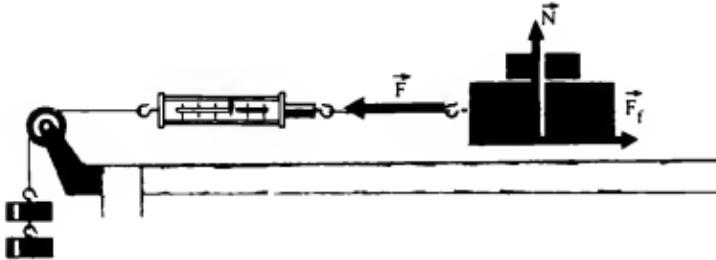


Fig. 103



Fig. 104



Fig. 105

the following relation for the magnitudes of these forces:

$$F_{f\max} = \mu N,$$

where  $\mu$  is the proportionality factor called the *coefficient of friction*.

**FRICTION DOES NOT ALWAYS HINDER MOTION.** It was mentioned earlier that friction is an obstacle for the beginning of motion. On the other hand, however, there exist situations when just static friction initiates motion. For example, it is just static friction  $F_1$  acting on the sole that imparts an acceleration to a walking person (Fig. 104). The sole does not slide backwards, hence friction between the sole and the ground is static. And if the sole is sliding, it is impossible to walk. The force  $F_2$  equal in magnitude to  $F_1$  and having the opposite direction imparts an acceleration to the Earth. In a similar way, the wheels of motorcars and other vehicles as if push off the Earth, and this pulling force is just static friction.

When the transmission belt in a belt drive makes the pulley rotate (Fig. 105), the force imparting an acceleration to the pulley ring is the static friction between the belt and the pulley.

?

1. A boy pushes a bookcase with a maximum possible effort but cannot shift it. Is Newton's second law, stating that a body acted upon by a force changes its velocity, violated here?
2. Is friction acting on a table standing on the floor?
3. When does static friction appear? What is its direction?
4. What is called the force of pressure?

## 5.7. Sliding Friction

It was established in the preceding section that if a force applied to a body parallel to the surface of its contact with another body is if only slightly stronger than the maximum static friction, the body acquires an acceleration and starts sliding over the surface of the other body. In this case too friction is acting on the body, but now it is *sliding friction*. Measurements show that this force is approximately equal in magnitude to the maximum static friction. The direction of sliding friction (henceforth, this force will be called just friction) is always opposite to that of the relative velocity of the bodies in contact. This is the most important property of friction.

*The direction of (sliding) friction is opposite to the direction of the velocity of a body relative to the body in contact with it.*

The acceleration imparted to a body by friction also has the direction opposite to that of the relative velocity of the body. This means that sliding friction always leads to the reduction of the relative velocity of the body.

Just as the maximum static friction, sliding friction is proportional to the force of pressure (and hence to the normal reaction  $N$ ) acting on a body:

$$F_f = \mu N.$$

The proportionality factor  $\mu$  in this formula is the same as in the expression for the maximum static friction.

The formula for friction shows that the coefficient  $\mu$  is equal to the ratio of the magnitudes of friction and the normal reaction:

$$\mu = \frac{F}{N}.$$

Normally, the coefficient of friction is less than unity. This means that friction is always smaller than the force of pressure.

The coefficient of friction characterizes simultaneously two bodies rubbing against each other rather than a body on which friction is acting. Its value depends on the materials of which rubbing bodies are made, on finishing the surfaces, their cleanliness, etc. Experiments revealed that friction is independent both of the area of surfaces in contact and of the relative position of the bodies. For example, the coefficient of friction between a skate and ice is the same for the entire ice road (of course, if the ice surface is uniform everywhere). Thus, friction is an exception to the rule according to which the force acting on a body depends on its position relative to the body with which it is interacting. It turns out that friction depends not on the position of a body but on its relative velocity. The velocity dependence of friction consists in that *a change in the direction of the velocity brings about a change in the direction of friction*.

The values of the coefficient of friction for some materials are compiled in the table.

Materials	Coefficient of friction
Wood over wood	0.25
Rubber over concrete	0.75
Leather belt over cast iron pulley	0.56
Steel over steel	0.20

These values of the coefficient of friction refer to nonlubricated surfaces. Lubrication considerably reduces friction. For example, when a lubricant is applied, steel slides over steel as easily as steel over ice does: the coefficient of friction amounts to only 0.04. Friction of two solid surfaces in contact (without a lubricant) is called *dry friction*.

**WHY DOES A LUBRICATION OF FRICTION SURFACES REDUCE THE COEFFICIENT OF FRICTION?** As a matter of fact, when solid bodies are moving in contact with fluids, there is also a force parallel to the contact surface and directed *against* the relative velocity of the body. In this respect, it resembles dry friction. This force is often called *fluid friction*. (It is also called the *drag force*.)

Fluid friction is considerably smaller than dry friction. It is well known that it is not difficult to push a raft off the shore for a person standing on it with a boathook. However, there is no use in trying to employ this method on the shore. This is why lubrication reduces friction (friction is no longer dry!).

There is no static friction in fluids. This means that even a very small force applied to a body in a fluid imparts an acceleration to it.

The absence of static friction in liquids is demonstrated by the following experiment. Let us put a small wooden bar on the surface of water in a wide vessel (Fig. 106). It is easy to set the bar in motion (to change its velocity) even by a very small force. It is sufficient to blow on it or push it with a paper strip. If, however, we put this bar on the table, it can be set in motion only by a sufficiently large force exceeding the maximum static friction.

In contrast to dry friction, the drag force in a fluid depends not only on the direction but also on the magnitude of the relative velocity of a body in the fluid. At a low velocity, the drag is proportional to it, while at a high velocity it is proportional to its square.

Besides, drag force greatly depends on the shape of a body.

Figure 107 represents three bodies having the same cross-sectional areas. If, however, these bodies move in a fluid at the same velocity, the largest drag force turns out to act on the plane discus (top figure) and the smaller drag, on the drop-shaped body (bottom figure).

The shape of a body for which drag is small is called *streamline shape*. Therefore, aircraft, motorcars and other vehicles moving at high speeds in fluids are given a streamline shape and their surfaces are thoroughly finished. This helps to reduce the drag force.

Concluding the section, it should be noted that the nature of friction is far from being studied completely.



Fig. 106

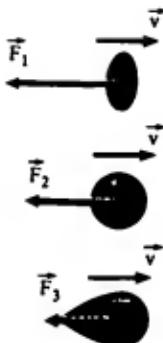


Fig. 107

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1. What is called sliding friction (dry friction)? What is its direction?
2. Give the definition of the coefficient of friction.
3. Why is it dangerous to drive a car on an icy road?
4. Which force should be applied to a body lying on a horizontal plane to initiate its motion over this plane?
5. The friction between the wheels of a bicycle and the ground is almost independent of velocity. At the same time, it is well known that the higher the velocity of the bicycle, the stronger muscular force should be applied by the cyclist to the bicycle pedals. Why is it so?
6. Should a streamline shape be given to spacecraft and to the rockets that bring them to space?
7. Why are not tractors and road rollers given a streamline shape?

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#### Exercise 16

1. Calculate the force required for pushing a 20-kg wooden bar over a wooden floor at a constant speed. What will be the motion of the bar if the force applied to it is larger than the one obtained in calculations?
2. A horse working for a long time develops a force of 600 N. What is the maximum load it can draw in sledges whose mass is 100 kg, if the coefficient of friction between the slides and the snow is 0.05? Assume that the shafts are parallel to the road.
3. A rubber bar is pressed against a vertical concrete wall by a spring. The elastic force of the spring is normal to the wall and equal to 100 N. Which force should be applied to the bar to initiate its motion?

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## Summary

All forces known in nature are manifestations of a few types of interaction. Forces considered in mechanics are the results of only two types of interaction, viz. electromagnetic and gravitational interaction.

Electromagnetic interaction is responsible for elastic forces and friction.

Elastic force emerges upon a deformation of a body as a result of displacement of some its parts relative to others. The projection of elastic force is determined by the following equation (Hooke's law):

$$(F_{\text{el}})_x = -kx.$$

The force of universal gravitation is a manifestation of gravitational interaction. The magnitude of this force is given by the following equation:

$$F = G \frac{m_1 m_2}{r^2}.$$

Elastic force and gravitational force depend on the mutual arrangement of interacting bodies, i.e. on the coordinates.

The force of attraction of bodies by the Earth near its surface is equal to  $mg$  and can be assumed constant if the distance from a body to the Earth's surface is small in comparison with the Earth's radius.

Friction emerges when bodies in contact are either at rest (static friction) or in motion (sliding friction). Friction is directed along the contact surface against the direction of the relative motion of the bodies in contact. Friction depends not on the coordinate of one body relative to the other but on their relative velocity.

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**THE SAME LAWS OF MOTION FOR ALL FORCES**

Using the laws of motion discovered by Newton and being able to measure or calculate forces, we can solve the fundamental problem of mechanics: to determine the acceleration from known forces and initial conditions, then to determine the velocity from its acceleration, and finally, the coordinates (position) of a body at any instant of time.

The situation when only one force (elastic force, friction or force of gravity) is acting on a body arises not frequently. In most cases a body is acted upon by several forces simultaneously. Then the acceleration is determined by the resultant of all the applied forces.

However, there are cases when although several forces are acting on a body, only one of them plays a significant role. The other forces either balance one another or are too small in magnitude.

We shall first consider precisely such cases.

## 6.1.

### **Motion of a Body Under the Action of Elastic Force**

Let us start with the situation when the initial velocity of a body is either zero or parallel to the elastic force.

It was shown earlier that the projection  $(F_{el})_x$  of the elastic force  $F_{el}$  onto the  $X$ -axis is equal to  $-kx$ . Consequently, the force  $F_{el}$  changes with the position of the body on which it is acting. It should be recalled that the elongation of a spring (or any other elastic body) is determined by the position of a body relative to the end of the undeformed spring.

What is the motion of a body under the action of such a varying force? We can find an answer to this question in an experiment.

Let us attach one end of a spring to a cart carrying a body of a large mass. The other end of the spring is fixed (Fig. 108). We pull the cart to the right by several centimetres and then release it. The cart starts moving periodically back and forth relative to its initial position. Such motion is called *vibrational*.

It is easier to observe vibrational motion of a body by suspending it from a spring (Fig. 109). Hauling the body downwards by several centimetres and releasing it, we initiate the vibrational motion of the body.

Using Newton's second law, we can find the position of the body at any instant of time. However, it is a difficult problem since the elastic force is

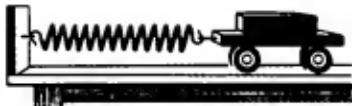


Fig. 108

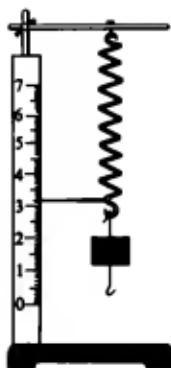


Fig. 109

a varying quantity. Vibrational motion will be considered in detail in the *Senior Physics 2*.

The motion of a body to which the initial velocity is imparted perpendicularly to the elastic force acting on the body is quite different.

A similar case was considered in Sec. 4.6 (see Fig. 81). It was shown that for such a mutual orientation of velocity and elastic force, the body moves in a circle.

Consequently, when the elastic force is at right angles to the initial velocity, it imparts to the body a centripetal acceleration and makes it move in a circle.

?

1. What is the motion of a body if an elastic force is the only force acting on it?
2. What is known about the acceleration of a body on which a varying force (e.g., an elastic force) is acting?
3. Does an elastic force applied to a body always lead to vibrational motion of the body?

#### Hometask

Watch the behaviour of a body being weighed on a spring balance. Does it reach the state of rest immediately?

## 6.2.

### Motion Under the Action of Force of Gravity: a Body Moves Along the Vertical

As long ago as at the end of 16th century, Galileo Galilei established that the free fall of a body is a uniformly accelerated motion. Moreover, he found that all bodies fall at the same acceleration. Later measurements showed that the magnitude of this

acceleration is  $9.8 \text{ m/s}^2$ . At that time, and even long after that, these were the facts established by observations and measurements but nevertheless quite enigmatic and unexplained.

Only Newton's laws of motion and of universal gravitation provided an explanation to these facts. Falling bodies have an acceleration since the force of gravity is acting on them. The acceleration of bodies falling on the Earth's surface is constant since the force of gravity is constant. Finally, the fact that all bodies are falling at the same acceleration irrespective of their mass is explained by the proportionality of the force of gravity (and in general the force of universal gravitation) to the mass of the body to which it is applied. This was discussed in Sec. 5.3.

Thus, under the action of the force of gravity, a body is in a uniformly accelerated motion so that the acceleration vector  $\vec{g}$  is directed downwards (downward direction is just defined as the direction of the vector  $\vec{g}$  at a given point), and its magnitude is  $9.8 \text{ m/s}^2$ .

It should be borne in mind that the acceleration of a falling body remains unchanged if we push the body downwards, imparting to it an initial velocity  $\vec{v}_0$ . In this case, the velocity will be increasing not from zero but from the value  $v_0$ .

The magnitude and direction of the acceleration also remain unchanged if a body is thrown upwards with a certain initial velocity. In all these cases, the trajectory of the body is a vertical straight line, and hence the motion of the body is rectilinear and uniformly accelerated.

While solving problems concerning this type of motion, it is convenient (although not necessary) to choose the Earth as a reference body and take the origin on its surface or at any point above or below the surface. The coordinate axis is usually directed upwards or downwards along the vertical. The height is denoted by  $h$ . Then (Fig. 110), the coordinate of a body is just its height above the reference point. In this case, the projection  $y - y_0$  of the displacement corresponds to the change in the height. Hence, the projection of the displacement is  $h - h_0$ , where  $h_0$  is the initial height ( $h_0 = y_0$ ).

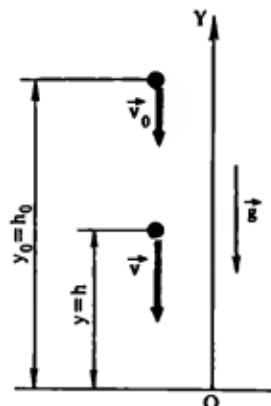


Fig. 110

The formulas for calculating coordinates (heights) and velocities do not differ from the formulas obtained in Secs. 2.2-2.3 for a rectilinear uniformly accelerated motion.

The coordinate (height) of a body is given by

$$y = h = h_0 + v_{0y} t + \frac{g_y t^2}{2}. \quad (6.2.1)$$

The velocity of a body at any instant of time is

$$v_y = v_{0y} + g_y t. \quad (6.2.2)$$

The velocity of a body at any point of the trajectory is

$$v_y^2 = v_{0y}^2 + 2g_y (h - h_0). \quad (6.2.3)$$

The projection  $g_y$  is positive if the  $Y$ -axis is directed downwards and negative if it has the upward direction. The projections  $v_{0y}$  and  $v_y$  are positive when velocities are parallel to the axis and negative when the velocities are antiparallel to it.

---

1. What is called a free fall of bodies?
2. What are the accelerations of a freely falling body and of a body thrown downwards?
3. What is the acceleration of a body thrown upwards? What is its magnitude and direction?
4. What is the difference between the acceleration imparted to bodies by the force of gravity and that imparted by other forces?
5. Why is the acceleration imparted to a body by the force of gravity constant and independent of its mass?
6. Would the motion of a body falling to the Earth from the height of several hundred or thousand kilometres be uniformly accelerated? Would the acceleration depend on the mass of the body in this case?

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## EXAMPLES OF SOLVING PROBLEMS

1. A body has fallen from the height of 100 m. Find the time of the fall of the body and its velocity at the moment it strikes the Earth's surface.

*Solution.* Let us choose as the origin of the  $y$ -coordinate (height) the point on the surface of the Earth and direct the  $Y$ -axis upwards (see Fig. 110). Then  $g_y = -g$ ,  $v_y = -v$  and  $v_{0y} = 0$  (the body fell and was not thrown!). Finally, at the moment the body touches the Earth its height  $h = 0$ .

The time of fall can be found with the help of formula (6.2.1) which assumes the form

$$0 = h_0 + 0 - \frac{gt^2}{2}.$$

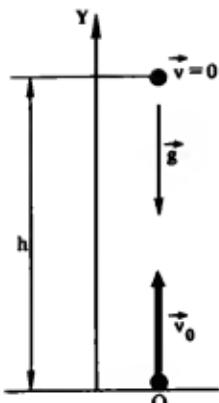


Fig. 111

Hence

$$t = \sqrt{\frac{2h_0}{g}},$$

$$t = \sqrt{\frac{2 \times 100 \text{ m}}{9.8 \text{ m/s}^2}} \simeq 4.5 \text{ s.} \quad (1)$$

The landing velocity can be calculated from formula (6.2.2) which in our case becomes

$$-v = 0 - gt \quad \text{or} \quad v = gt,$$

$$v = 9.8 \frac{\text{m}}{\text{s}^2} \times 4.5 \text{ s} = 44 \frac{\text{m}}{\text{s}}.$$

2. What is the maximum height of a body thrown upwards at the initial velocity  $v_0 = 44 \text{ m/s}$ ? Calculate the time required to reach this height.

*Solution.* As in the previous problem, we direct the  $Y$ -axis upwards (Fig. 111). In this case,  $v_{0y} = v_0$  and  $g_y = -g$ . In the uppermost point, the velocity of the body  $v = 0$ . Then Eq. (6.2.2) becomes

$$0 = v_0 - gt.$$

Hence we find the time of ascent:

$$t = \frac{v_0}{g}, \quad t = \frac{44 \text{ m/s}}{9.8 \text{ m/s}^2} \simeq 4.5 \text{ s.} \quad (2)$$

Since  $h_0 = 0$ , the maximum height can be calculated with the help of formula (6.2.3). Using the conditions of the problem, we have

$$0 = v_0^2 - 2gh,$$

whence

$$h = \frac{v_0^2}{2g}, \quad h = \frac{(44 \text{ m/s})^2}{2 \times 9.8 \text{ m/s}^2} \simeq 100 \text{ m.} \quad (3)$$

A comparison of Problems 1 and 2 shows that the time required for a body to fall from a certain height is equal to the time of ascent to the same height if the initial velocity of the body thrown upwards is equal to the final velocity of the falling body. This is not surprising. Both the falling body and the body thrown upwards experience the action of the same force, viz. the force of gravity  $F_g$  which imparts the same acceleration  $\bar{g}$  to them.

### Exercise 17

While solving these problems, assume that the resistance offered by air can be neglected.

1. A stone was falling to the bottom of a ravine during 40 s. What was the depth of the ravine?
2. How long would a load falling from the Ostankino television tower (540 m height) take to reach the ground? What would be its velocity at this moment?
3. How much time will it take for a body that started to fall from a state of rest to cover a distance of 4.9 m? What is its velocity at the end of motion?
4. A boy standing on the brim of a 180 m cliff dropped a stone and a second later he threw downwards the second stone. What was the initial velocity imparted to the second stone if both stones touched the ground simultaneously?
5. A body falls freely from the height of 20 m above the ground. What is its velocity at the moment it strikes the Earth? At what height does its velocity amount to a half of this value?
6. Coloured plate I shows consecutive positions of a freely falling ball in every 0.1 s. Using the figure, find the free-fall acceleration if the initial velocity of the ball was zero. The scale is such that a square size is  $4.9 \times 4.9$  cm.
7. An arrow is shot from a bow vertically upwards at a velocity of 30 m/s. What is its maximum height?
8. A body thrown from the ground upwards has fallen in 8.0 s. Find the height of its ascent and the initial velocity.
9. A ball shot from a spring pistol located at 2.0 m above the ground flies upwards at the initial velocity of 5.0 m/s. Determine its maximum height and its velocity at the moment it falls to the ground. What is the time of the ball's flight? What is its displacement during the first 0.2 s of the flight?
10. A body is thrown upwards at a velocity of 40 m/s. At what height will it be in 3 and 5 s? What will be its velocities at these moments? Assume that  $g = 10 \text{ m/s}^2$ .
11. Two bodies are thrown upwards with different initial velocities. The height reached by one body is four times larger than that for the other body. What is the ratio of their initial velocities?

12. A body thrown upwards flies past the window at a velocity of 12 m/s. What will be its velocity at the moment it passes the same window during its downward motion?

## 6.3.

### Motion Under the Action of Force of Gravity: Initial Velocity of the Body is at an Angle to the Horizontal

We often deal with motion of bodies having acquired an initial velocity not parallel to the force of gravity but at a certain angle to it (or to the horizontal). Such a body is said to be thrown at an angle to the horizontal. For example, when a sportsman pushes a shot or throws a discus or a lance, he imparts to these objects an initial velocity of this kind. In artillery firing, barrels of guns have a certain angle of elevation so that a fired shell also acquires an initial velocity at an angle to the horizontal.

We shall assume that the resistance offered by air can be neglected. What is the motion of a body in this case?

Figure 112 represents a stroboscopic photograph of a ball thrown at an angle of  $60^\circ$  to the horizontal. Connecting consecutive positions of the ball by a smooth line, we obtain the trajectory of motion of the ball. It is the well-known curve—parabola.

It was known even to Galilei that a body thrown at an angle to the horizontal moves along a parabola. But the explanation to this effect also was given only by Newton's laws of motion and the law of universal gravitation.

Suppose that a body is thrown from a certain point at a certain initial velocity  $v_0$  directed at an angle  $\alpha$  to the horizontal. We take for the origin the point from which the body is thrown and direct the  $X$ - and  $Y$ -axes as shown in Fig. 113. For the reference time we take the instant when the body was thrown. It can be seen from the figure that the projection of the initial velocity  $v_0$  onto the  $X$ - and  $Y$ -axes are  $v_0 \sin \alpha$  and  $v_0 \cos \alpha$  respectively,

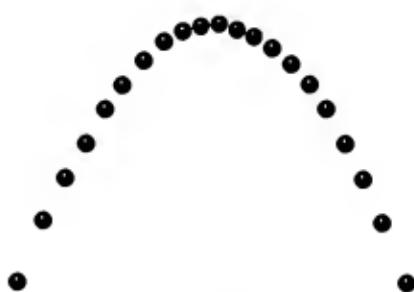


Fig. 112

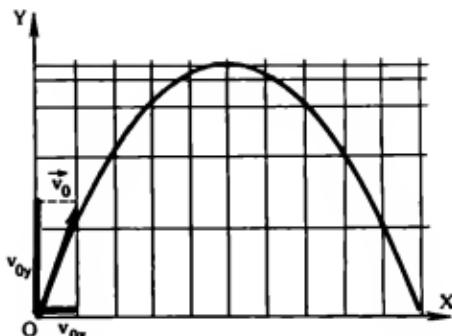


Fig. 113

where  $v_0$  is the magnitude of the initial velocity  $\bar{v}_0$ :

$$v_{0y} = v_0 \sin \alpha, \quad v_{0x} = v_0 \cos \alpha.$$

Since only the force of gravity is acting on the body, which is directed downwards, only the  $y$ -projection of the velocity  $\bar{v}$  will change in the motion, while the  $x$ -projection of the velocity will remain unchanged.

Hence, the  $x$ -coordinate of the body changes with time in the same way as in uniform rectilinear motion:

$$x = v_{0x} t. \quad (6.3.1)$$

On the other hand, the  $y$ -coordinate varies as in a uniformly accelerated rectilinear motion:

$$y = v_{0y} t + \frac{g_y t^2}{2}. \quad (6.3.2)$$

In order to plot the trajectory of this motion, we must substitute into Eqs. (6.3.1) and (6.3.2) successively, increasing values of time  $t$  and calculate the  $x$ - and  $y$ -coordinates for each instant of time  $t$ . Using these coordinates, we must plot the points representing consecutive positions of the body. The smooth curve drawn through these points is the trajectory we are interested in. It is shown in Fig. 113. Having this curve, we can find the value of a coordinate for a certain value of the other coordinate.

**A BODY IS THROWN ALONG THE HORIZONTAL.** A body can be thrown in such a way that its initial velocity  $\bar{v}_0$  is directed along the horizontal ( $\alpha = 0$ ). For example, this is the case when a body is separated from a horizontally flying aeroplane. The shape of the trajectory along which the body will move can be easily found. For this purpose, let us consider again Fig. 113 showing the trajectory of a body thrown at an angle to the horizontal. At the upper point of the parabola, the velocity of the body just has the horizontal direction. As we know, having passed this point, the body moves along the right-hand branch of the parabola. Obviously, any body thrown at a certain initial velocity  $\bar{v}_0$  in the horizontal direction will also move along the branch of the parabola (Fig. 114).

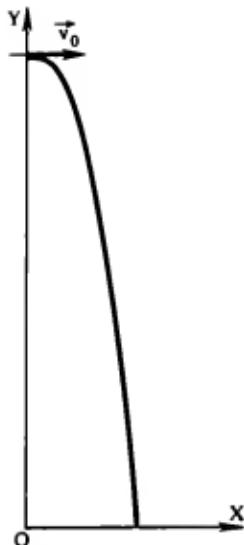


Fig. 114



Fig. 115

The trajectory of a body thrown along the horizontal or at an angle to it can be visualized in a simple experiment. A vessel filled with water is placed at a certain height above the table and is connected by a rubber tube having a spout with a tap (Fig. 115). Discharged jets directly show the trajectories of water particles. Changing the angle of the jet discharged, we see that the maximum horizontal range is attained at an angle of  $45^\circ$ .

We have analyzed several examples of motion of bodies under the action of the force of gravity. They show that in all cases a body moves with the acceleration  $\bar{g}$  imparted to it by the force of gravity. This acceleration is completely independent of whether or not the body also moves in the horizontal direction. We can even say that in all these cases the body is in a free fall.

For this reason, for example, a bullet shot from a gun in the horizontal direction falls on the ground simultaneously with another bullet dropped by chance by the gunman at the moment of shot. However, the bullet dropped by chance falls at the feet of the gunman while the one shot from the gun falls in several hundred metres from the gunman.

Coloured plate *Ib* represents a stroboscopic photograph of two balls one of which falls vertically and the other acquires a horizontal velocity at the moment the first ball starts falling. The photograph shows that at the same instants (flashes of light) both balls are at the same height and naturally they reach the ground simultaneously.

While considering the motion of a body thrown along the horizontal or at an angle to it, we assumed that the body experiences the action of the force

of gravity alone. Actually, this is not so. Besides the force of gravity, the force of resistance (friction) is always exerted on the body in air. This force reduces the velocity of the body.

Due to the drag force, the actual trajectory of a body thrown along the horizontal or at an angle to it is not a parabola but a more complex curve.

?

1. What are common features in motion of bodies thrown vertically, horizontally and at an angle to the horizontal?
2. What is the trajectory of a body thrown at an angle to the horizontal?
3. Which force is acting on a body thrown at an angle to the horizontal?
4. Can the motion of a body thrown at an angle to the horizontal be considered as uniformly accelerated?
5. What is the acceleration of a body thrown at an angle to the horizontal? What is the direction of this acceleration?  
*Hint.* Answering these questions, assume that friction can be neglected.

## EXAMPLES OF SOLVING PROBLEMS

1. A shell flies from a cannon at an angle  $\alpha$  to the horizontal with an initial velocity  $v_0$ . Find (a) the time of flight of the shell; (b) its maximum height and (c) horizontal range.

*Solution.* The motion of a body thrown at an angle to the horizontal is described by Eqs. (6.3.1) and (6.3.2).

Since  $v_{0x} = v_0 \cos \alpha$ ,  $v_{0y} = v_0 \sin \alpha$  and  $g_y = -g$ , we have

$$x = v_0 t \cos \alpha,$$

$$y = v_0 t \sin \alpha - \frac{gt^2}{2}.$$

(a) At the end of the shell flight, the coordinate  $y = 0$ . Consequently, the time of flight can be found from the equation for the  $y$ -coordinate:

$$0 = v_0 t \sin \alpha - \frac{gt^2}{2}.$$

Solving this equation, we obtain

$$t_1 = 0, \quad t_2 = \frac{2v_0 \sin \alpha}{g}.$$

The value  $t_1 = 0$  corresponds to the beginning of flight (at this instant, the  $y$ -coordinate is also equal to zero), while  $t_2$  is the time of flight:

$$t_{\text{flight}} = \frac{2v_0 \sin \alpha}{g}.$$

Due to the symmetry of the parabola, the time of ascent to its vertex is equal to a half of the time of flight, i.e.

$$t_{\text{asc}} = \frac{v_0 \sin \alpha}{g}.$$

(b) The maximum height  $h_{\text{max}}$  is the value of the  $y$ -coordinate obtained by substituting the value of the time of ascent into the expression for the  $y$ -coordinate:

$$h_{\text{max}} = v_0 \sin \alpha \frac{v_0 \sin \alpha}{g} - \frac{g}{2} \left( \frac{v_0 \sin \alpha}{g} \right)^2.$$

Simplifying this expression, we get

$$h_{\text{max}} = \frac{v_0^2 \sin^2 \alpha}{2g}. \quad (1)$$

(c) The horizontal range  $l$  is equal to the value of the  $x$ -coordinate obtained by substituting the time of flight for  $t$  into the expression for  $x$ :

$$l = v_0 \cos \alpha \frac{2v_0 \sin \alpha}{g} = \frac{2v_0^2 \sin \alpha \cos \alpha}{g}.$$

We can easily find the angle  $\alpha$  for which the range is maximum. It is well known that  $2 \sin \alpha \cos \alpha = \sin 2\alpha$ . Hence the expression for the horizontal range can be written as follows:

$$l = \frac{v_0^2 \sin 2\alpha}{g}. \quad (2)$$

It follows that the range is maximum when  $\sin 2\alpha = 1$ . This corresponds to  $2\alpha = 90^\circ$  or  $\alpha = 45^\circ$ .

It should be noted that the horizontal range and the maximum height in air are always smaller than the corresponding values obtained from (1) and (2) since air offers a resistance to motion.

2. When an aeroplane flying horizontally at a speed  $v_0 = 720$  km/hr is at a height  $h = 3920$  m a load is dropped from it. How far from the place above which it was released will the load touch the ground?

*Solution.* At the moment of separation from the plane the load has the initial velocity  $v_0$  which has the horizontal direction and is equal in magnitude to the velocity of the aeroplane. This instant should be taken as the time reference point, while for the origin of coordinates we take the point at which the load is dropped. We direct the  $X$ -axis along the horizontal and the

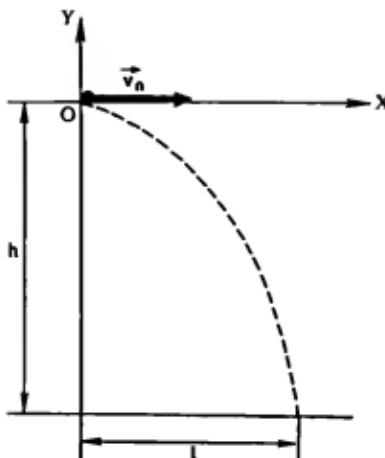


Fig. 116

$y$ -axis, vertically upwards (Fig. 116). The motion of the load is described by the well-known equations

$$x = v_0 t \cos \alpha, \quad y = v_0 t \sin \alpha - \frac{gt^2}{2}.$$

In this problem  $\alpha = 0$ , and hence  $\sin \alpha = 0$ ,  $\cos \alpha = 1$ . Then the equations describing the motion of the load dropped from the aeroplane take the form

$$x = v_0 t, \quad y = - \frac{gt^2}{2}.$$

The horizontal range  $l$  is the value of the  $x$ -coordinate obtained by substituting for the time  $t$  the time of fall for the load. This time can be determined from the equation for the  $y$ -coordinate. At the moment of landing,  $-y = h$  and hence

$$-h = - \frac{gt^2}{2}.$$

We can find the time of fall for the load:

$$t = \sqrt{\frac{2h}{g}}.$$

Consequently,

$$l = v_0 \sqrt{\frac{2h}{g}},$$

$$l = 200 \frac{\text{m}}{\text{s}} \sqrt{\frac{2 \times 3920 \text{ m}}{9.8 \text{ m/s}^2}} \simeq 5600 \text{ m.}$$

---

### Exercise 18

1. A ball is thrown at an angle of  $30^\circ$  to the horizontal at the initial velocity of 10 m/s. Find the maximum height, the time and the horizontal range of the ball's flight.
2. A bullet is shot in the horizontal direction at an initial velocity of 800 m/s. Calculate the descent of the bullet in the vertical direction during its flight if the distance to the target is 600 m.

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### Hometask

Show that formulas (6.2.2) and (6.2.3) describing the motion of a body thrown vertically upwards can be obtained as a particular case of formulas for the motion of a body thrown at an angle  $\alpha$  to the horizontal if we assume that this angle is  $90^\circ$ .

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## 6.4.

### Weight of a Body. Weightlessness

**WEIGHT OF A BODY.** It should be recalled (see *Junior Physics*, Sec. 29) that *weight is the force exerted by a body on a support or suspender due to its attraction to the Earth*.

Why does such a force appear? What are its magnitude and direction?

Let us consider, for example, a body suspended from a spring whose other end is fixed (Fig. 117 right). The body experiences the action of the force of gravity  $\bar{F}_g = m\bar{g}$ , directed downwards. Hence it starts to fall down entraining the lower end of the spring. The spring is thus deformed, which gives rise to the elastic force  $\bar{F}_{el}$  of the spring. It is applied to the top of the body and is directed upwards. Hence, when the body falls, its top will "lag behind" its other parts which are not acted upon by the elastic force. As a result, the body is deformed as well (this is shown in Fig. 117 on a magnified scale). Another elastic force, viz. the elastic force of the deformed body, emerges. It is applied to the spring and is directed downwards (see Fig. 117 left). It is just this force that is called the weight of the body. It is denoted by  $\bar{P}$ . In accordance with Newton's third law, the two elastic forces are equal in magnitude and have opposite directions.

After several vibrations, the body comes to rest. This means that the force of gravity  $m\bar{g}$  is equal in magnitude to the elastic force of the spring (see Fig. 117b). The weight  $\bar{P}$  of the body is also equal to the same force. Thus, in this example the weight of the body is equal in magnitude to the force of gravity  $m\bar{g}$ :

$$\bar{P} = m\bar{g}.$$

This does not mean, however, that the weight of a body and the force of gravity applied to it are identical forces. The force of gravity is

a *gravitational* force applied to the body, while the weight is the elastic force applied to the *suspender*.

If a body is placed on a support instead of being suspended (Fig. 118), the support also experiences the action of the force emerging in a similar way and also called the weight.

**WEIGHTLESSNESS.** Let us suppose that a spring with a load suspended from it (or a spring balance) is held in the hand (Fig. 119). The weight of the body can be seen on the scale of the spring balance. If the hand holding the spring balance is at rest, the balance indicates that the weight  $P$  of the body is equal to the force of gravity  $mg$ . Now suppose that the spring is released so that it falls freely with the load. It would be easily noted that the pointer of the balance in this case stands against the zero division, indicating zero weight of the body. It is not surprising. In free fall both the balance and the load move at the same acceleration equal to  $g$ . The lower end of the spring is not entrained by the load but just follows it so that the spring is not deformed. Hence there is no elastic force acting on the load. This means that the load is not deformed either and does not act on the spring. The weight has vanished. The load is said to become *weightless*.

Weightlessness is explained by the property of the force of universal gravitation (and, in particular, the force of gravity) to impart the same acceleration to all bodies (in the case under consideration, to the load and spring) (see Sec. 5.3). Therefore, each body moving under the action of the *force of gravity alone* (or, in general, the force of universal gravitation) is in the state of *weightlessness*. Any freely falling body is just under such conditions. It should, however, be borne in mind that the fact that the

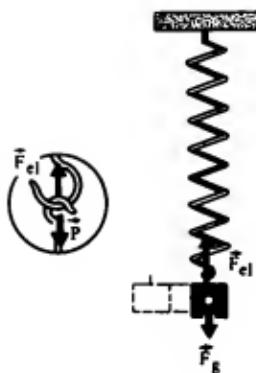


Fig. 117

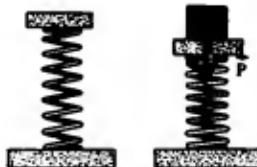


Fig. 118

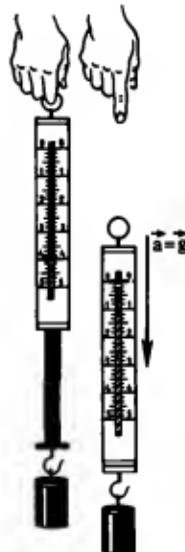


Fig. 119

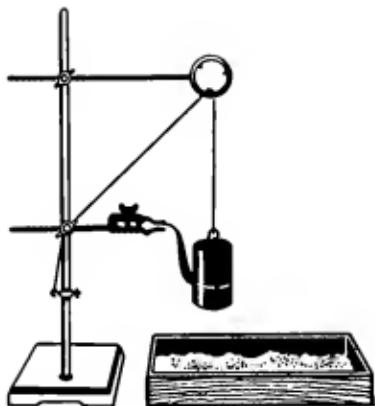


Fig. 120

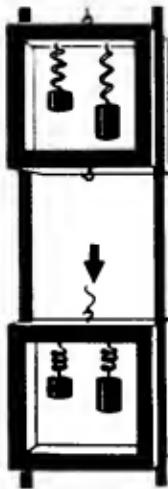


Fig. 121

pointer of the balance is at zero in the experiment considered above does not mean that the force of gravity has disappeared. It is weight that has vanished, i.e. the force exerted by the load on the suspender. As regards the force of gravity acting on the balance as well as on the load, it is present and causes the free fall.

Weightlessness is not a rare state for people on the Earth. A jumper is in this state from the moment of its separation from the ground to landing, as well as a swimmer springing from a tower from the moment of separation from it to the instant he touches water. Even a runner is in the state of weightlessness during short lapses of time between the instants he touches the ground by a foot.

The emergence of weightlessness in free fall can be observed in the following experiment.

A strip of paper is placed between the weights of a composite load (Fig. 120), its free end being tightly clamped in the tang of a holder. If the load is released slowly, the strip is stretched, i.e. deformed, and then ruptured. Hence it follows that the paper strip was clutched by the loads quite tightly. The ruptured strip is then replaced by another, intact one and the load is let fall freely. The new paper strip remains intact in the tang of the holder. This experiment shows that in free fall the weight does not exert pressure on the support, i.e. the load becomes weightless during the free fall.

Another experiment is illustrated in Fig. 121. Two different loads are suspended from two identical springs in a frame which can slide along two guiding rods. Naturally, the springs are stretched to different extents by the loads. If, however, we burn the thread that holds the frame, the latter will fall freely, and it can be seen that the deformations of the springs disappeared: the loads have become weightless!

---

1. What is called the weight of a body?
2. What is the difference between the weight and the force of gravity acting on a body?
3. A body is resting on a support. Which forces are acting on this body and on the support?
4. When is a body said to be in a state of weightlessness? What is the general reason behind weightlessness?
5. Is a body in a state of weightlessness when thrown upwards? Friction in air should be neglected.
6. Is a body in a state of weightlessness when thrown along the horizontal? What about a body thrown at an angle to the horizontal? Friction of air should be neglected.

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## 6.5.

### Weight of a Body Moving with an Acceleration

**WEIGHT OF A BODY CAN BE LESS THAN THE FORCE OF GRAVITY.** Let us now consider the case when a body attached to a spring balance is moving with an acceleration relative to the Earth but is not in a free fall. To realize such a motion, we can, without releasing the spring balance, lower it abruptly, imparting to it a certain downward acceleration  $\bar{a}$  (Fig. 122). It can be easily seen that the pointer of the balance will go up. This means that the weight of the body has become less than it was when the balance and the body were at rest. Why has the weight become smaller?

The body is acted upon by (1) the downward force of gravity  $\bar{F}_g = m\bar{g}$  and (2) the upward elastic force  $\bar{F}_{el}$  of the balance spring. These two forces together impart the acceleration  $\bar{a}$  to the body. According to Newton's second law, we have

$$\bar{F}_{el} + m\bar{g} = m\bar{a}. \quad (6.5.1)$$

The three vectors in this equation are parallel to the Y-axis which we directed downwards (see Fig. 122). Hence formula (6.5.1) for the projections of these vectors onto the Y-axis assumes the form

$$(F_{el})_y + mg_y = ma_y. \quad (6.5.2)$$

Figure 122 shows that the vectors  $\bar{g}$  and  $\bar{a}$  have the same direction as the Y-axis. Consequently, their projections are positive and equal in magnitude to the vectors themselves:  $g_y = g$  and  $a_y = a$ . The vector  $\bar{F}_{el}$ , however, is directed opposite to the Y-axis, and hence its projection is negative:  $(F_{el})_y = -F_{el}$ . Therefore, formula (6.5.2) can be written as follows:

$$-F_{el} + mg = ma \quad \text{or} \quad F_{el} = mg - ma.$$

The weight  $P$  is equal in magnitude to the elastic force  $F_{el}$  (according to

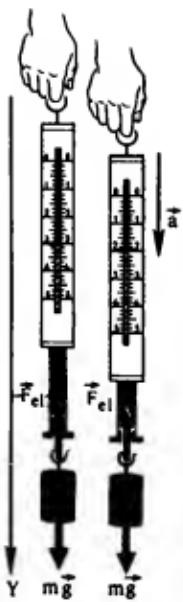


Fig. 122

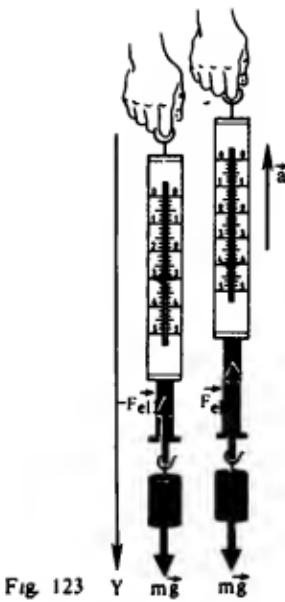


Fig. 123

Newton's third law), and hence

$$P = mg - ma. \quad (6.5.3)$$

This formula shows that if  $a < g$ , the weight of the body is less than the force of gravity  $mg$ , i.e. less than the weight of the body at rest.

If a body (of course, together with a support or suspender) is moving with an acceleration having the same direction as the acceleration due to gravity, its weight is less than that in a state of rest.

It should be recalled once again that we speak of a decrease in the weight and not in the force of gravity.

**THE WEIGHT OF A BODY CAN ALSO BE LARGER THAN THE FORCE OF GRAVITY.** If a spring balance with a body suspended from it is abruptly raised, thus imparting to it an upward acceleration  $\ddot{a}$  (Fig. 123), the pointer of the balance goes down, indicating an increase in the weight of the body. The above reasoning is valid for this case too, the only difference being that the projection of the vector  $\ddot{a}$  onto the  $Y$ -axis is now negative, and hence the formula for the magnitude of the weight becomes

$$P = mg + ma. \quad (6.5.4)$$

The weight of the body is now larger than the force of gravity  $mg$ , i.e. larger than the weight of the body at rest.

If a body is moving with an acceleration whose direction is opposite to that of the free-fall acceleration, its weight is larger than that in a state of rest.

An increase in the weight of a body due to its accelerated motion is called *overload*.

The weight of a body increases or decreases not only when the body moving with an acceleration is suspended from a spring or a spring balance. The same applies to any suspender or support.

Let us consider several examples in which the weight of a body changes with an acceleration.

1. A motorcar moving over a convex bridge (Fig. 124) is lighter than the same car resting on the same bridge.

Indeed, the motion over the convex bridge is the motion along a segment of a circle. Hence, the motorcar is moving with a centripetal acceleration whose magnitude is given by

$$a = \frac{v^2}{r},$$

where  $v$  is the car's velocity and  $r$  is the radius of curvature. At the instant when the car is at the uppermost point of the bridge, this acceleration is directed vertically downwards. It is imparted to the car by the resultant of the force of gravity  $\bar{F}_g = m\bar{g}$  and of the normal reaction  $\bar{N}$  of the bridge.

The equation expressing Newton's second law in the vector form is written as follows:

$$m\bar{g} + \bar{N} = m\bar{a}.$$

We direct the  $Y$ -axis upwards and write this equation for the projections of the vectors onto this axis:

$$mg_y + N_y = ma_y.$$

Obviously,

$$g_y = g, \quad N_y = -N \quad \text{and} \quad a_y = a = \frac{v^2}{r}.$$

It follows that

$$mg - N = m \frac{v^2}{r},$$

whence

$$N = m \left( g - \frac{v^2}{r} \right).$$

The weight  $\bar{P}$  of the motorcar (i.e. the force with which it presses against the bridge) is, according to Newton's third law, equal in magnitude and opposite in direction to the normal reaction  $\bar{N}$  of the bridge. Consequently,

$$P = N = m \left( g - \frac{v^2}{r} \right), \quad P < mg.$$

Similarly, the weight of the passengers in the motorcar moving across a convex bridge decreases.

2. A pilot taking his plane out of a dive (Fig. 125) experiences overload in the lower part of the trajectory. Indeed, in this part of the trajectory, the plane is moving in a circle with a centripetal acceleration directed vertically

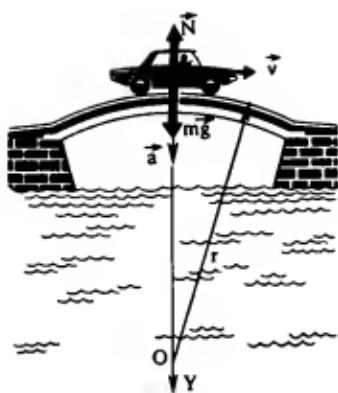


Fig. 124

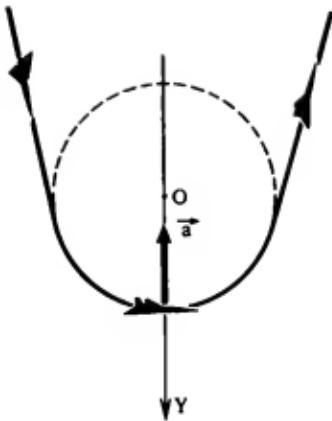


Fig. 125

upwards to the centre of the circle. The magnitude of this acceleration is

$$a = \frac{v^2}{r}.$$

Its projection onto the downward vertical axis is negative:

$$a_y = -a = -\frac{v^2}{r}.$$

Consequently, the weight of the pilot, i.e. the force with which he acts on the support (seat) will be, in accordance with formula (6.5.3), defined as

$$P = m(g + a) = m\left(g + \frac{v^2}{r}\right), \text{ i.e. } P > mg.$$

Thus, the weight of the pilot is greater than the "normal" weight, equal to the force of gravity  $mg$ , by the quantity  $mv^2/r$ . If at the end of the dive the centripetal acceleration  $v^2/r$  is  $n$  times larger in magnitude than the free-fall acceleration  $g$  ( $v^2/r = ng$ ), the weight of the pilot is

$$P = m(g + ng) = mg(n + 1),$$

i.e. is  $n + 1$  times as large as the "normal" weight of the pilot.

The weight of pilot's intestines also increases in overload, as well as the force with which they act on each other and on his skeleton. This causes painful reaction and may become hazardous to health at a too high overload. Well-trained pilots can withstand the overload up to  $10 mg$  (the overload is normally expressed not in  $mg$ 's but in  $g$ 's; it is said that the overload is, for example,  $10 g$ ).

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9

1. In what way does the weight of a body change during its accelerated motion?
2. Does the weight of a body change if it moves with an acceleration in the horizontal direction?
3. What will be the change in the weight of a cosmonaut at the start of the rocket bringing his spacecraft to its orbit?
4. What is the change in the weight of a cosmonaut during the deceleration of the landing spacecraft?
5. What happens to the weight of a pilot performing a wingover when he is at the lower and upper points of the acrobatic manoeuvre?

---

### Exercise 19

1. A crane uniformly displaces a concrete 500-kg plate, (a) vertically upwards, (b) horizontally and (c) vertically downwards. What is the force of gravity acting on the plate and what is its weight in each case?
2. A 100-kg load is fixed on the bottom of a mine cage. What is the weight of this load if the cage (a) moves vertically upwards with an acceleration of  $0.3 \text{ m/s}^2$ , (b) moves uniformly, (c) moves downwards with an acceleration of  $0.4 \text{ m/s}^2$ , and (d) falls freely?
3. What is the decrease in the weight of a motorcar at the uppermost point of a convex bridge? The radius of curvature of the bridge is 100 m, the mass of the car is 2000 kg and its velocity is 60 km/hr.
4. Calculate the weight of a body having a mass of 1000 g at the pole and on the equator, assuming that the radius of the Earth is 6400 km.

---

## 6.6.

### Artificial Earth's Satellites. Orbital Velocity

It was shown in Sec. 6.3 that a body acquiring at the height  $h$  above the Earth an initial velocity  $\bar{v}$  in the horizontal direction (i.e. parallel to the Earth's surface) describes a parabola. Moving along this parabola, the body falls to the ground.

While considering this motion, the surface of the Earth was assumed to be plane. Such a simplification is justified for comparatively low velocities  $\bar{v}$  for which the displacement of the body in the horizontal direction is small (Fig. 126a).

**THE EARTH RUNS FROM UNDER THE BODY.** As a matter of fact, the Earth is a sphere. For this reason, as the body moves along its trajectory, the Earth's surface moves away from it to a certain distance (Fig. 126b). We can choose such a magnitude of the velocity  $\bar{v}$  for the body that the distance separating the body from the surface of the Earth due to curvature is equal

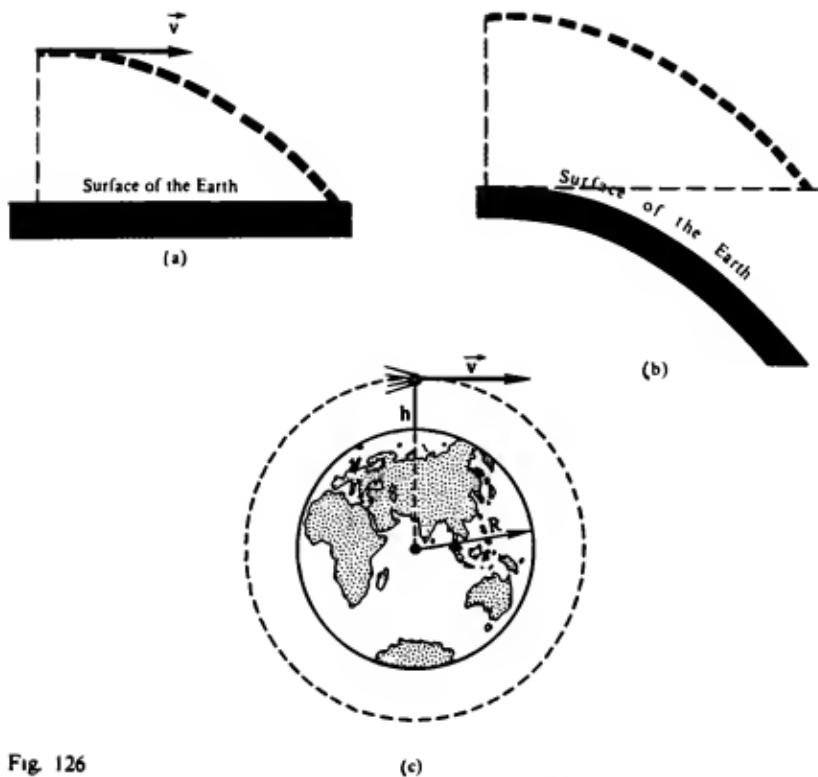


Fig. 126

to the distance to which the body approaches the Earth due to its attraction. Then the body moves at a certain constant distance  $h$  above the Earth's surface, i.e. in a circle of radius  $R + h$ , where  $R$  is the radius of the globe (Fig. 126c). What is this velocity?

**ARTIFICIAL EARTH'S SATELLITE.** Since the body is moving uniformly in a circle, the magnitude of its acceleration is

$$a = \frac{v^2}{R + h}.$$

This acceleration is imparted to the body by the gravitational force of the Earth, whose magnitude is

$$F = G \frac{Mm}{(R + h)^2}.$$

Here  $M$  is the mass of the Earth and  $m$  is the mass of the body. According to Newton's second law, we have

$$a = \frac{F}{m} = G \frac{M}{(R + h)^2}.$$

Consequently,

$$\frac{v^2}{R+h} = G \frac{M}{(R+h)^2},$$

whence

$$v = \sqrt{G \frac{M}{R+h}}. \quad (6.6.1)$$

This means that if the body acquires the velocity having the horizontal direction and specified by formula (6.6.1), it moves in a circle around the Earth, i.e. becomes its artificial satellite.

**ORBITAL VELOCITY.** A body of any mass may only become an Earth's satellite if a sufficient velocity is imparted to it. Let us calculate this velocity for a satellite launched near the Earth's surface ( $h = 0$ ):

$$v = \sqrt{G \frac{M}{R}}.$$

It should be recalled that  $G(M/R^2) = g$ . Consequently,

$$G \frac{M}{R} = gR.$$

Hence

$$\underline{v = \sqrt{gR}}.$$

Substituting  $g = 9.8 \text{ m/s}^2$  and  $R = 6.4 \times 10^6 \text{ m}$  into this formula, we obtain

$$v = \sqrt{9.8 \frac{\text{m}}{\text{s}^2} \times 6.4 \times 10^6 \text{ m}} \simeq 8 \times 10^3 \text{ m/s} \simeq 8 \text{ km/s}.$$

This is the velocity which should be imparted to a body in the horizontal direction near the Earth's surface for it to become a satellite moving in a circular orbit instead of falling to the Earth's surface. This velocity is called the *orbital velocity*.

Eight kilometres per second – it is about 29 000 kilometres per hour! Of course, it is not easy to impart such a huge velocity to a body. It was only in 1957 when Soviet scientists managed for the first time in the history of mankind to impart the orbital velocity to a body whose mass was about 84 kg with the help of a high-power rocket. This body has become Earth's first artificial satellite.

Satellites are orbiting the Earth under the action of a single force, viz. the force of universal gravitation which imparts the same acceleration to the satellite and all the objects in it. As was mentioned in Sec. 6.4, in this case the very concept of weight loses its meaning since a body and its "support" do not deform each other and cannot press on each other. This means that all bodies in a satellite, including cosmonauts, are in a state of weightlessness.

?

1. What must be the direction of the velocity of a body at the moment it is brought to a circular orbit to become an artificial Earth's satellite?
2. What is the direction of the acceleration of an artificial Earth's satellite?
3. Can we assume that the motion of an artificial satellite is uniformly accelerated?
4. The Soviet cosmonaut A. Leonov was the first to perform a space walk, leaving its spacecraft. Was he in a state of weightlessness during this time?

### Exercise 20

1. Calculate the period of revolution of an Earth satellite at a height of 300 m.
2. Calculate the orbital velocity for the height above the Earth which is equal to its radius.
3. At what height above the Earth is the orbital velocity equal to 6 km/hr?
4. At what height above the Earth's surface should an artificial Earth's satellite be launched so that its period of revolution be 24 hr?

## 6.7.

### Motion of a Body Under the Action of Friction

Sliding friction differs from all other forces in that its direction is opposite to the direction of the relative velocity of motion of rubbing bodies.

Hence it follows that the acceleration imparted by friction to a body moving over a fixed surface is directed against the relative velocity. This means that *friction leads to a decrease in the magnitude of the velocity of the body*.

If no forces, except friction, are acting on a body sliding over a fixed surface, it ultimately stops. Let us consider this case which is often encountered in practice. Suppose that a certain unexpected obstacle appears in front of a moving train. A driver switches off the engine and applies the brake. Starting from this moment, the train is acted upon only by a constant friction, since the force of gravity is balanced by the normal reaction of the rails, and the air resistance is small. In a certain time  $t$  the train, having traversed a certain distance  $l$  called the *braking distance*, stops. Let us find the time  $t$  required for the train to stop and the braking distance  $l$  covered during this time.

Under the action of the force  $\bar{F}_{fr}$ , the train moves with an acceleration  $\bar{a} = \bar{F}_{fr}/m$ .



Fig. 127

Let us choose the  $X$ -axis so that its positive direction coincides with the direction of the train velocity (Fig. 127). The friction  $\bar{F}_{fr}$  and the acceleration  $\bar{a}$  imparted by it are directed against the  $X$ -axis, and hence the projections of these vectors onto the  $X$ -axis are negative and equal to the magnitudes of these vectors taken with the minus sign. This means that the magnitude of acceleration  $a = -a_x = F_{fr}/m$ . But  $a_x = (v_x - v_{0x})/t$ , where  $v_x$  and  $v_{0x}$  are the projections of the final and initial velocity vectors. Both of them are positive, i.e.  $v_x = v$  and  $v_{0x} = v_0$ . Thus,

$$a = -\frac{v - v_0}{t}.$$

We are interested in the time  $t$  elapsing from the beginning of braking (when the train velocity was  $v_0$ ) to the halt ( $v = 0$ ). In this case,

$$a = \frac{v_0}{t} \quad \text{and} \quad t = \frac{v_0}{a} = \frac{mv_0}{F_{fr}}.$$

**THIS IS IMPORTANT TO EVERYBODY.** Let us calculate the braking distance. This distance is the  $x$ -projection of the vector  $\bar{s}$  of train displacement over the time  $t$ . It can be calculated by the formula

$$s_x = \frac{v_x^2 - v_{0x}^2}{2a_x}.$$

In the case under consideration,  $s_x = s$ ,  $v_x = 0$ ,  $v_{0x}^2 = v_0^2$  and  $a_x = -a = -F_{fr}/m$ . Consequently,

$$l = s = \frac{mv_0^2}{2F_{fr}}.$$

This formula shows that the distance covered to the halt is proportional to the square of the initial velocity. If this velocity is doubled, the braking distance increases fourfold. This should be remembered by engine drivers, car drivers and in general by all those who drive vehicles. It is also important for pedestrians crossing a street with heavy traffic. Time and space are required for stopping moving bodies.

?

1. What is the direction of an acceleration imparted to a body by friction?
2. Can we assume that the motion under the action of friction is uniformly accelerated?

3. Give examples of motion without friction in nature.
4. Can a body be stopped instantaneously by braking?
5. Which quantities determine the braking distance of a moving body? What will be the change in this distance if each of these quantities is doubled?
6. In order to reduce the braking distance (the distance covered by a body to a halt), either friction must be increased or the velocity of motion must be decreased. Which of these methods is more effective?

---

### Exercise 21

1. What was the velocity of a propeller-driven sledge if it covered the distance of 250 m after its engine had been switched off?  $\mu = 0.02$ .
2. A car driver switched off the motor and quickly applied the brake at a speed of 72 km/hr. What time will the car move to a halt if  $\mu = 0.60$ ? What will be the braking distance?

---

### Hometask

Give examples of motion of bodies under the action of friction alone.

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## 6.8.

### Motion of a Body Under the Action of Several Forces

Earlier in this chapter we analyzed the motion of bodies under the action of only one force. This force can be caused by elasticity, gravity or friction. In actual practice, however, we virtually never deal with such motions. Besides elastic and gravitational forces, a body always experiences the action of friction.

**FALLING BODY IN A FLUID.** This is an interesting example of a rectilinear motion of a body under the action of two forces. In this case, the force of gravity and the resistance of a fluid (drag force) described in Sec. 5.7 are acting on the body.

If we ignore all other forces, we can assume that at the moment when the body starts to fall ( $v = 0$ ), only the force of gravity  $F_g$  is acting on it. The drag force is zero. As soon as the body starts moving, a drag force (i.e. fluid friction) appears, which increases with the velocity and is directed opposite to it.

Since the force of gravity remains constant, while the drag force having the opposite direction increases with the velocity of the body, in a certain time the two forces will be equal in magnitude. At this moment, the resultant of the two forces becomes equal to zero. The acceleration of the body will thus also vanish, and the body will move at a constant velocity. For example,

a parachutist starts to move at a constant speed soon after the beginning of the jump. Snowflakes and rain drops also fall at a constant velocity.

If a body is falling in a fluid, one more force should be taken into account. This is the Archimedian force (buoyancy) directed upwards. But since this force is constant and independent of the velocity it does not prevent the body from falling uniformly in the fluid.

The uniform motion of a body in a fluid can be observed in the following simple experiment. A glass tube of about 1 m long is completely filled with water or glycerin. Then a steel ball is dropped into the liquid (Fig. 128). It can be easily seen that the ball is moving through the liquid at a constant velocity. This can be proved by painting divisions along the tube and measuring the distances covered by the ball during equal time intervals counted by a metronome.

**HOW TO SOLVE PROBLEMS IN MECHANICS WHEN SEVERAL FORCES ARE ACTING ON A BODY?** First of all, it should be recalled that the force  $\vec{F}$  appearing in the equation for Newton's second law

$$\vec{F} = m\vec{a}$$

is the vector sum of all the forces applied to a body. Thus, when solving a problem, one should first plot on the figure the vectors of all these forces and the acceleration of the body (if its direction is known). Then, having chosen the direction of coordinate axes, one should find the projections of all the vectors onto these axes. Finally, one should write the equations of Newton's second law for the projections onto each axis and solve the obtained scalar equations simultaneously.

If the motion of a system of bodies is being considered, the equation for Newton's second law is applied to every body of the system and then the obtained equations are solved simultaneously.

?

1. What is the difference between the drag force acting on a falling body in a fluid and the sliding friction?
2. Why does the motion of a parachutist become uniform in a certain time after the opening of the parachute?
3. Formulate Newton's second law for the case when several forces are acting on a body.

#### EXAMPLES OF SOLVING PROBLEMS

1. A bar  $A$  of mass  $m$  is moving downwards along an inclined plane with a slope  $\alpha$  (Fig. 129). The coefficient of friction between the bar and the plane is  $\mu$ . Find the acceleration of the bar.

*Solution.* Three forces are acting on the bar: the force of gravity  $\vec{F}_g = m\vec{g}$ , the normal reaction of the support  $\vec{N}$  (elastic force) and friction  $\vec{F}_{fr}$ . The directions of these forces are indicated in the figure. The three forces together

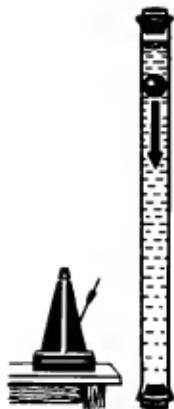


Fig. 128

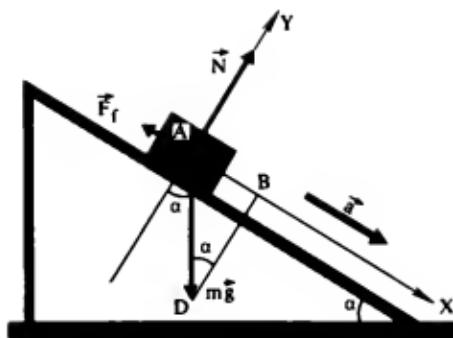


Fig. 129

impart to the bar an acceleration  $\bar{a}$  directed along the inclined plane.<sup>11</sup>

Let us direct the coordinate axes  $X$  and  $Y$  so that they are parallel and perpendicular to the inclined plane respectively. Newton's second law in vector form is written as

$$m\bar{a} = m\bar{g} + \bar{N} + \bar{F}_{fr}. \quad (1)$$

In order to write this equation in scalar form, we must find the projections of the vectors onto the  $X$ - and  $Y$ -axes. Let us start with the  $x$ -projections. The projection  $a_x$  of the acceleration vector  $\bar{a}$  onto the  $X$ -axis is positive and equal in magnitude to  $\bar{a}$  (vector  $\bar{a}$  is parallel to the  $X$ -axis):  $a_x = a$ . The projection of the force of gravity  $m\bar{g}$  is positive and, as can be seen from  $\Delta ABD$  (see Fig. 129), is  $mg \sin \alpha$ . The projection of the friction  $\bar{F}_{fr}$  is negative and is  $-F_{fr}$ . Finally, the projection of the normal reaction  $\bar{N}$  is equal to zero, since this vector is perpendicular to the  $X$ -axis:  $N_x = 0$ .

Newton's second law for the projections onto the  $X$ -axis, which are expressed in terms of the magnitudes of the vectors, has the form

$$ma = mg \sin \alpha - F_{fr}. \quad (2)$$

Let us now find the  $y$ -projections. The projection of the acceleration vector  $\bar{a}$  onto the  $Y$ -axis is zero (vector  $\bar{a}$  is perpendicular to the  $Y$ -axis):  $a_y = 0$ . The projection of the force of gravity  $m\bar{g}$  onto the  $Y$ -axis is negative and is  $-mg \cos \alpha$ , as can be seen from Fig. 129. The projection of the normal reaction  $\bar{N}$  is positive and equal to it in magnitude:  $N_y = N$ . Finally, the projection of the friction force  $\bar{F}_{fr}$  is equal to zero.

<sup>11</sup> In order to simplify Fig. 129, we assumed that all the three forces are applied to the same point, viz. the centre of the bar. Actually, the forces  $\bar{F}_{fr}$  and  $\bar{N}$  are applied to its base.

In this case, the equation for Newton's second law has the form

$$0 = N - mg \cos \alpha, \quad (3)$$

whence

$$N = mg \cos \alpha.$$

It is well known (see Sec. 5.7) that the magnitude of the friction force is  $\mu N$ . Hence  $F_{fr} = \mu mg \cos \alpha$ . Substituting this expression for the friction force into (2), we get

$$ma = mg \sin \alpha - \mu mg \cos \alpha.$$

Cancelling  $m$ , we find the required acceleration of the bar:

$$a = g (\sin \alpha - \mu \cos \alpha).$$

It follows from this formula that  $a$  is smaller than the acceleration of free fall.

If there is no friction ( $\mu = 0$ ), the acceleration of a body sliding along an inclined plane is equal in magnitude to  $g \sin \alpha$ , i.e. it is still less than  $g$ . That is why inclined planes are widely used in practice, since they make it possible to reduce the acceleration of a body sliding up or down along them.

2. A thread is passed over a fixed pulley with loads of mass  $m_1$  and  $m_2$  attached to it ( $m_1 > m_2$ ). Find the acceleration of the loads, assuming that the masses of the thread and pulley are small in comparison with  $m_1$  and  $m_2$  and that there is no friction in the pulley.

*Solution.* We direct the Y-axis vertically upwards (Fig. 130).

If we let the system alone, the load having mass  $m_1$  will start moving downwards, while the load of mass  $m_2$  will move upwards. Let us find the acceleration (whose magnitude will be the same for the two bodies if we ignore the elongation of the thread:  $a_1 = a_2 = a$ ). For this purpose, we write the equation of Newton's second law for each load.

The left load experiences the action of the force of gravity  $\vec{F}_{g1} = m_1 \vec{g}$  and the tension of the thread  $\vec{F}$  (elastic force). The projection of the force of

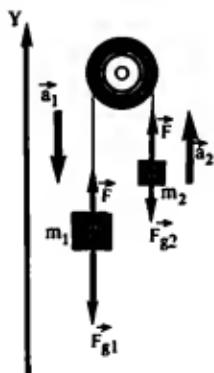


Fig. 130

gravity onto the  $Y$ -axis is equal to the magnitude of the vector  $m_1\bar{g}$  with the opposite sign:  $m_1g_y = -m_1g$ . The  $y$ -projection of the force  $\bar{F}$  is equal to the magnitude of the vector  $\bar{F}$ :  $\bar{F}_y = F$ . The projection of the acceleration  $\bar{a}_1$  is equal in magnitude to vector  $\bar{a}_1$  with the opposite sign:  $a_{1y} = -a_1 = -a$ . The equation of Newton's second law for the left load has the form

$$-m_1a = -m_1g + F. \quad (1)$$

The right load is acted upon by the force of gravity  $\bar{F}_2 = m_2\bar{g}$  and the tension of the thread  $\bar{F}$  (the same as that acting on the left load if the mass of the thread is assumed to be zero). The  $y$ -projection of the force of gravity is equal in magnitude to the vector  $m_2\bar{g}$  with the opposite sign:  $m_2g_y = -m_2g$ . The  $y$ -projection of the force  $F$  is equal in magnitude to the vector  $\bar{F}$ :  $F_y = F$ . The projection of the acceleration  $\bar{a}_2$  is equal to the magnitude of the acceleration vector  $\bar{a}_2$ :  $a_{2y} = a_2 = a$ .

The equation of Newton's second law for the right load has the form

$$m_2a = -m_2g + F. \quad (2)$$

Subtracting termwise (1) from (2), we obtain

$$m_2a - (-m_1a) = -m_2g + F - (-m_1g) - F,$$

or

$$(m_1 + m_2)a = (m_1 - m_2)g.$$

Hence

$$a = \frac{m_1 - m_2}{m_1 + m_2} g.$$

Since the difference in masses of the loads is less than their sum, the acceleration  $a$  is less than the free-fall acceleration. Pulleys are sometimes used to make a body fall with an acceleration less than  $g$ . This principle is used in the design of counterweights of lifts and other hoisting machines.

### Exercise 22

- Find the geometrical sum of the forces applied to the bar on an inclined plane (see Fig. 129) by plotting. What is the direction of the resultant  $\bar{F}$  of these forces relative to the inclined plane?
- A bar slides from the top of an inclined plane of a 20-cm height. Find the velocity of the bar at the foot of the plane, ignoring friction.
- A sledge slides down the hill of length 10 m during 2 s. Find the slope of the hill, ignoring friction.
- A 50-kg body is on an inclined plane whose height is 5 m and length 10 m. A horizontal force  $\bar{F}$  of 300 N is acting on the body (Fig. 131). Find the acceleration of the body, neglecting friction.

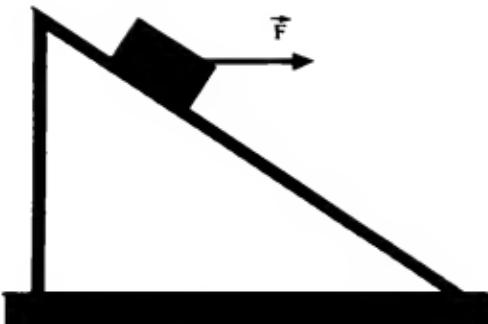


Fig. 131

5. Calculate the acceleration of a body sliding down an inclined plane having equal height and base, if the coefficient of friction between the body and plane is 0.20.
6. A ball suspended from a thread rotates in a horizontal plane, completing one revolution during 0.50 s. With which force does it act on the thread which makes it rotate? The mass of the ball is 200 g.

#### Hometask

Prove that for a uniform motion of a body along an inclined plane  $\mu = \tan \alpha$ , where  $\mu$  is the coefficient of friction and  $\alpha$  is the slope.

## 6.9. Motion on Bends

It was shown more than once that in order to make a body move in a circle, the force applied to it should be directed towards the centre of the circle. If several forces are acting on the body, the resultant of these forces must be directed to the centre of the circle.

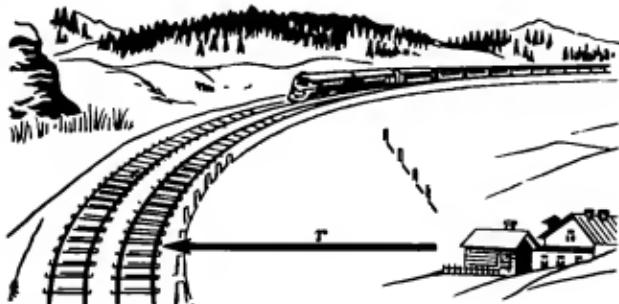


Fig. 132

By way of an example, let us consider the motion of a railway car on a rounding of a horizontal track (Fig. 132).

As long as the train is moving over a rectilinear track at a constant velocity  $\bar{v}$ , every car, naturally, experiences the action of the force of gravity which is balanced by the upward elastic force (normal reaction) of the rails. As to friction, it is balanced by the pulling force of the locomotive.

The car now reaches the rounding of the track. Here it turns and starts moving along the arc of a circle. What is the force that makes the car change the direction of the velocity, i.e. move with an acceleration? This force is the elastic force (normal reaction) exerted on the wheels of the car by the rail.

The wheels of railway carriages have a flange which touches the rail not from above but from the side (Fig. 133). As long as the carriage moves over a rectilinear track, the flange does not play any role and only the part of the wheel which touches the rail from above is deformed. Having passed point  $A$  (Fig. 134), the wheel, continuing on its way, acts through the flange on the rail and deforms it on one side. The rail is bent outwards (naturally, the flange itself is also deformed). This gives rise to an elastic force  $\bar{F}$  directed normally to the lateral surface of the rail. It is this force that makes the carriage move in a circle. If the wheels of the carriage had no flanges, this force could not appear, and the carriage would go off the rails.

The magnitude of the acceleration of the car moving at a velocity  $\bar{v}$  over the rounding of radius  $r$  is  $v^2/r$ . Hence, according to Newton's second law, the magnitude of the elastic force  $\bar{F}$  exerted on the flange (and hence on the car) by the deformed rail is

$$F = \frac{mv^2}{r},$$

where  $m$  is the mass of the car.

The deformation of the rail just attains the value at which the elastic force due to this deformation imparts the acceleration  $v^2/r$  to the car. This deformation is very small and cannot be noticed by naked eye (dashed line in Fig. 134).

For reducing the wear of rails and flanges, it is necessary to reduce the friction between them, i.e. reduce the force of pressure of the rail on the



Fig. 133

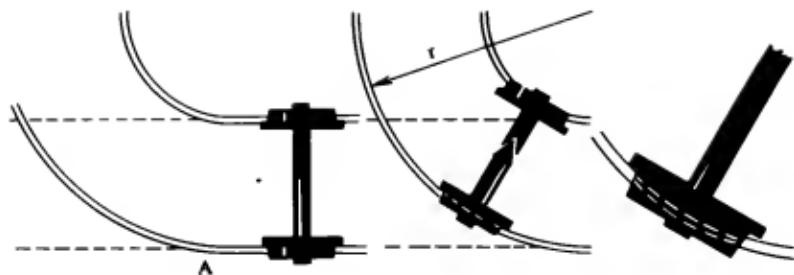


Fig. 134

flange. For this purpose, the railroad bed at roundings is made slightly inclined to the centre of the rounding (Fig. 135). In this case, the normal reaction  $\bar{N}$  of the rails (elastic force) does not balance the force of gravity  $\bar{F}_g$ . Their resultant  $\bar{F}_1$  is directed approximately to the centre of the rounding. This, obviously, "facilitates" the turn since the magnitude of the elastic force  $\bar{F}$  exerted by the rail on the flange becomes smaller. Indeed, now the same centripetal acceleration  $v^2/r$  is imparted to the car by two forces:  $\bar{F}$  and  $\bar{F}_1$ . Therefore, for a small angle we can write

$$\frac{v^2}{r} = \frac{\bar{F} + \bar{F}_1}{m},$$

whence

$$\bar{F} = \frac{mv^2}{r} - \bar{F}_1.$$

This expression shows that the magnitude of the force acting on the flange is reduced by  $\bar{F}_1$ . Therefore, the wear of the rail and flange is smaller.

The wheels of a motorcar have no flanges. When the car makes a turn on a highway, a centripetal acceleration is imparted to it by the dry friction between the tires and the asphalt surface (see coloured plate II).



Fig. 135

?

1. In what direction should a force be applied to a body so that its rectilinear motion is transformed into a curvilinear motion (at the bend)?
2. In this section we have considered the motion of a railway carriage at the bend of the track. The resultant of which forces imparts a centripetal acceleration to the carriage?
3. The resultant of which forces imparts a centripetal acceleration if there is no banking on the track?
4. Can sliding friction impart a centripetal acceleration to a body?
5. Why is it dangerous to take a turn on an ice-covered road?

### Exercise 23

1. A motorcar moving at a speed of 108 km/hr has to describe a turn of radius 50 m. Can it be safely done without reducing the speed, if the static friction between the wheels and asphalt surface is 4000 N and the mass of the car is 1000 kg? What is the maximum speed that ensures safety?
2. A train is moving over a bend of radius 500 m. The width of the track is 1.524 m. The outer rail is 12 cm above the inner one. Find the train velocity on the bend at which the flanges of the wheels exert no pressure on the rails.

## 6.10.

### Conditions of Translatory Motion of Bodies. Centre of Mass and Centre of Gravity

While studying the motion of bodies under the action of various forces, we paid no attention so far to the fact that bodies have dimensions. Determining acceleration of bodies, we assumed that they are material points.

Such a simplification is valid if a body is in translatory motion. However, it should be established at which point of the body a force must be applied for its accelerated motion to be translatory.

Let us make an experiment. We take a broad ruler, attach a thread to its end at point *A* and pull the thread with a certain force  $\vec{F}$  in the direction normal to the ruler axis (Fig. 136). The ruler turns, different points of the ruler covering different distances and moving with different velocities. In other words, they move in different ways, and the ruler is not in a translatory motion.

Let us now change the direction of the force: we pull the ruler along its longer side to the right (Fig. 137). The ruler is moving so that the velocities and displacements of all its points are the same. Thus, the ruler is in a translatory motion.



Fig. 136

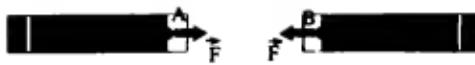


Fig. 137



Fig. 138

If the force  $\vec{F}$  is not balanced by other forces, the body moves with an acceleration. It can be easily seen that if the thread is fixed at point  $A$ , there exists only one straight line along which the force  $\vec{F}$  should be directed to cause an accelerated translatory motion of the ruler. A force acting along any other straight line causes the ruler to turn.

We can reverse the direction of the force by attaching the thread at point  $B$  (Fig. 138). The motion of the ruler will again be translatory. This means that only the position of the straight line along which the force acts (the line of force) is important.

Let us now fix the thread at any other point of the ruler, e.g. at point  $C$  (Fig. 139) and again change directions of tension of the thread (several directions are shown in the figure as straight lines emerging from point  $C$ ). Once again, we see that the ruler is in a translatory motion only if the force acts in a certain direction. This direction is marked by a red line in the figure. At all other directions of the force applied at point  $C$ , the ruler always turns.

Fixing the thread to other points of the ruler, we can be certain that for each point there exists only one direction of the force at which the ruler translates without rotation. Figure 140 shows the directions of forces applied at different points of the ruler to make it translate. Experiments show that the straight lines along which these forces act converge at the same point  $O$ .

**CENTRE OF MASS.** Similar experiments with other bodies lead us to an important conclusion that for each body there exists a point of intersection of lines of forces making the body perform an accelerated *translatory* motion. This point is called the centre of mass. Any force acting along the straight line that misses the centre of mass causes a rotation of the body.

*The centre of mass of a body is the point at which the force setting the body in an accelerated translatory motion is applied*

In the experiment with the ruler we can easily see that the centre of mass of the ruler coincides with the intersection of its diagonals. This, however, is the case only if the ruler is homogeneous (made of the same material), has a regular shape and the same thickness everywhere. If, for example, half the ruler is made, say, of wood and the other half is made of steel, the centre of mass would lie somewhere on the "steel" half, i.e. closer to the part having the larger mass.

The centre of mass of a body may turn out to be outside it. For example, a translatory motion of a homogeneous ring (Fig. 141) is possible only when the forces applied to it have the radial direction. Naturally, the lines of action of these forces converge at the geometrical centre of the ring. This is the point where its centre of mass is.

If different parts of the ring are made of different materials, the centre of

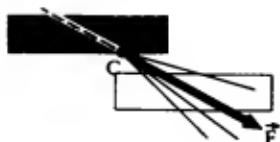


Fig. 139

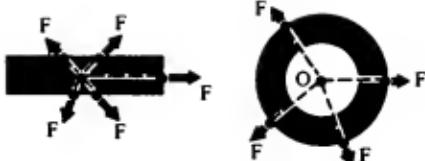


Fig. 140

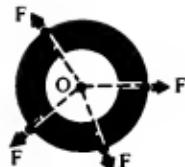


Fig. 141

mass may not coincide with its geometrical centre. In this case, the centre of mass should be found experimentally. There exist some methods of calculating the centre-of-mass coordinates, but they are cumbersome and even inapplicable in some cases.

But what for do we need the position of the centre of mass? As a matter of fact, if a body is in a translatory motion under the action of one or several forces, this means that this force or the resultant of all forces passes through the centre of mass of the body. *In this case, the centre of mass moves as if the entire mass of the body were concentrated in it and all the forces acting on the body were applied to it.*<sup>11</sup> Therefore, if a body is in a translatory accelerated motion, this means that the resultant of forces acting on it passes through its centre of mass as if the entire mass of the body were concentrated at this point and the acceleration of the body were the acceleration of its centre of mass. Thus, *instead of the motion of a body, we consider the motion of a material point, i.e. its centre of mass*. We proceeded in this way, without stipulating it, in the previous chapters of this book.

**CENTRE OF GRAVITY.** The motion of a body under the action of the force of gravity is a particular case of translatory motion (if, of course, the body has not been set in rotation before it began to fall). The force of gravity acts on all points of the body. If the body is in a translatory motion under the action of all these forces, this means that their resultant passes through the centre of mass of the body in any position. For this reason, the centre of mass is often called the *centre of gravity*.

## 6.11. Are the Laws of Newtonian Mechanics Always Valid? (Motion from Different Points of View)

Summing up what has been considered above, we must first of all pay attention to the fundamental ideas of mechanics.

The *first idea* consists in that if no force acts on a body or if the resultant of all forces is zero, the body is at rest or in motion at a velocity of constant magnitude and direction. If, however, the body moves with an acceleration, the motion necessarily occurs under the action of a force. The state of rest or

<sup>11</sup> It can be shown that this is valid for any kind of motion



Fig. 142

uniform rectilinear motion are impossible if there is a force, while an accelerated motion cannot proceed without a force.

According to the *second fundamental idea* of mechanics, a force can act on a body only in the presence of another body, large or small, remote or close.

There may or may not be a direct contact with this body. But behind every force there is necessarily "hidden" a body or several bodies. In other words, any force is material in origin.

These two ideas constitute the essence of the Newtonian mechanics.

Are these fundamental statements always correct? To answer this question, let us imagine an experiment which can be actually carried out. (This question was partially considered in Sec. 4.1.)

A man is standing on a flatcar with front and rear walls, which is attached to a locomotive. Suppose that the floor of the flatcar is made of a very rigid and smooth material, and the man is on rollers that can move with a very small friction (Fig. 142). Assume further that another man at the station watches him. Now, the flatcar starts off, i.e. begins to move with *an acceleration*.

**THE POINT OF VIEW OF THE STATIONARY OBSERVER.** The observer at the station sees that after the flatcar has started off, the man on it remains at rest and the floor of the flatcar just moves from under him. Knowing the laws of mechanics, the observer would say that this is what should be expected. The man on the flatcar remains at rest since the forces acting on him—the force of gravity and the elastic force of the flatcar—are directed vertically and balance each other. And only when the rear wall touches the man, he starts moving together with the flatcar. This is also in agreement with the laws of mechanics: the moving wall coming in contact with the man interacts with him and is deformed. As a result, an elastic force emerges, which imparts to the man an acceleration equal to that of the flatcar.

THE POINT OF VIEW OF THE MAN ON THE FLATCAR. The man on rollers perceives the situation quite differently. All of a sudden he realizes that he starts to move *relative to the flatcar* towards its rear wall with a certain acceleration. From his point of view, this contradicts the laws of mechanics. He will just be at a loss if he tries to find out which body has imparted the acceleration to him, since he will not find such a body.

Which of the observers is right?

Obviously, the personality of an observer does not play any role here. The reference systems relative to which the motion is observed cause the difference. The observer on the station speaks about motion relative to the Earth which he assumed to be a fixed reference system. On the other hand, the man on the flatcar has in mind the motion relative to the reference system attached to the flatcar. This reference system is moving with an acceleration relative to the Earth. The matter is just in the accelerated motion of one reference system, viz. the flatcar, relative to another system, viz. the Earth.

*The laws of Newtonian mechanics hold only under the condition that the motion is considered relative to inertial reference systems.*

It should be recalled that inertial reference systems are the systems in which bodies do not acquire an acceleration in the absence of forces (the man on the flatcar is at rest and the wall of the flatcar approaches him). If bodies in such systems acquire an acceleration, this means that forces *due to other bodies act* (the rear wall of the flatcar has touched the man and he starts to move with an acceleration together with the platform).

However, in the reference systems fixed to the flatcar, the laws of mechanics are invalid. Relative to this system, the man moves with an acceleration when no other bodies act on him. When the force actually emerges (the elastic force of the rear wall), the man stops. The reason behind the violation of Newton's laws in this system is its accelerated motion relative to the reference frame in which these laws hold, i.e. relative to the Earth. Indeed, as soon as the flatcar has gained speed and starts moving uniformly, the man on rollers moves relative to the flatcar without an acceleration. The laws of mechanics come to force.

If the laws of mechanics are valid for a motion relative to one reference frame, they remain valid also for a motion relative to any other reference frame that moves uniformly in a straight line relative to the first frame.

There is a countless number of such systems. In all inertial reference frames, the laws of motion are the same. This is the essence of the *Galilei relativity principle*.

All reference systems which move with an acceleration relative to an inertial system are called *non-inertial* systems since the law of inertia as well as Newton's second and third laws are not valid for them.

?

1. A load (pendulum) is suspended by a thread from the ceiling of a railway carriage. What happens to the pendulum during a deceleration of the car? How is this phenomenon explained by (a) an observer standing on a platform and (b) an observer in the car?

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2. Is it possible to determine the velocity of a steamer and its acceleration if a passenger in a cabin with closed porthole observes the motion of a load suspended by a thread from the ceiling of the cabin?

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#### Hometask

1. Invent a device which, when attached to a body, would allow us to measure its acceleration.
2. Consider an example similar to that described in this section when the flatcar is uniformly rotating.

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## Summary

Any problem in mechanics is solved with the help of Newton's laws if, besides initial coordinates and velocity, the forces applied to the body are known (in other words, if we know how these forces depend on coordinates and velocity). It should be borne in mind that the *force or resultant of several forces determines not the velocity of the body (its magnitude and direction) but its acceleration*. For this reason, bodies do not necessarily move in the direction of the force. The trajectory of motion of a body is determined not only by the forces applied to it but also by the initial conditions, viz. the magnitude and direction of the initial velocity of the body.

Motion of bodies can be treated as motion of material points only in the case of *translatory motion*. The body is in a translatory motion if and only if the line of action of the resultant of all the forces passes through the *centre of mass* of the body. Otherwise, the rotation of the body about a certain axis takes place in addition to its translation.

If we consider the motion of a body relative to a non-inertial reference system (viz. the reference system moving with an acceleration relative to any inertial system), the laws of Newtonian mechanics turn out to be invalid. The body moves relative to a non-inertial reference system with an acceleration which is not due to forces applied to it, and may move uniformly in the presence of forces.

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# 7

## FUNDAMENTALS OF STATICS (EQUILIBRIUM OF BODIES)

### WHAT DOES STATICS STUDY?

It was shown earlier that Newton's laws allow us to find accelerations acquired by bodies due to forces applied to them.

It is often important to know, however, the conditions under which bodies experiencing the action of various forces do not acquire accelerations. Such bodies are said to be in equilibrium state. In particular, this is the state of bodies at rest.

It is very important for practical purposes to know the conditions under which bodies remain at rest, for example, for constructing buildings, bridges, various supports, suspenders, for manufacturing engines, instruments, and so on.

For example, the tower of the Ostankino TV centre in Moscow cannot be allowed to acquire an acceleration under the action of wind and to move from its foundation. It is Newton's laws that allow us to find the conditions ensuring equilibrium and above all, the state of rest of a body.

The part of mechanics which studies equilibrium of rigid bodies is called *statics*.

It is well known that each body can be in translatory motion and, besides, can rotate or turn about a certain axis. Obviously, neither translatory nor rotary motion<sup>11</sup> of the body should change in equilibrium. In particular, if it is required that the body should be at rest, it must neither translate nor rotate (turn) about an axis.

Let us consider conditions of equilibrium of bodies for these two possible types of motion separately.

### 7.1. Equilibrium of Bodies in the Absence of Rotation

For translatory motion, we can consider the motion of only one point of the body, viz. its centre of mass. In this case, we assume that the entire mass of the body is concentrated at the centre of mass to which the resultant of all the forces acting on the body is applied. It follows from Newton's second law that the acceleration of this point is zero if the geometrical sum of all the forces applied to it (resultant) is equal to

<sup>11</sup> In rotational motion, all points of a body describe concentric circles around a point through which the axis of rotation passes.

zero. This is just the condition of equilibrium for a body in the absence of rotation.

*The necessary condition of equilibrium for a body that can translate (without rotation) is that the geometrical sum of forces applied to the body should be equal to zero.*

If, however, the geometrical sum of the forces is zero, the sum of projections of the vectors of these forces onto any axis also vanishes. Therefore, the condition of equilibrium can be formulated as follows.

*For a nonrotating body to be in equilibrium, it is necessary that the sum of the projections of the forces applied to it onto any axis is equal to zero.*

For example, a body to which two equal forces acting along the same straight line in opposite directions are applied (Fig. 143) is in equilibrium. Figure 144 illustrates an experiment that can be carried out in a school laboratory.

An equilibrium state is not necessarily a state of rest. It follows from Newton's second law that when the resultant of forces applied to a body vanishes, the body can be in a uniform rectilinear motion. In this motion, the body is also in equilibrium. For example, a parachutist reaches an equilibrium state when he starts falling at a constant velocity.

The forces shown in Fig. 143 are applied to the body not at the same point. It was shown, however, that it is the line along which the force is acting that is important rather than the point of its application. The motion of the body or its equilibrium state does not change when the point of application of a force is displaced along the line of its action. For example, it is obvious that nothing changes if the car shown in Fig. 145 is pushed instead of being pulled (Fig. 146).

If the resultant of forces acting on a body differs from zero, an additional force equal and opposite to the resultant is required for the body to be in equilibrium.



Fig. 143



Fig. 144



Fig. 145



Fig. 146

?

1. What is the meaning of the expression: "a body (or system of bodies) is in equilibrium"?
2. Several forces are applied to a body, such that their resultant differs from zero. What should be done to bring the body to an equilibrium state?
3. What are the conditions of the equilibrium for bodies in translatory motion?
4. Does the equilibrium necessarily imply a state of rest?
5. If the geometrical sum of forces applied to a body is zero, what is the algebraic sum of the projections of these forces onto a certain axis?

### EXAMPLE OF SOLVING A PROBLEM

How can we keep in equilibrium a boat acted upon by the river flow and the wind blowing from the bank (Fig. 147)?

*Solution.* Let us find the resultant  $\vec{F}$  of the forces  $\vec{F}_1$  and  $\vec{F}_2$  due to the wind and water flow by using the parallelogram rule. The diagonal of the parallelogram constructed on the forces as sides gives the magnitude and direction of the resultant  $\vec{F}$ . In order to keep the boat in equilibrium, the force  $\vec{F}_y$  equal and opposite to the resultant force direction should be applied to it. This force may be, for example, the elastic force of the rope whose one end is fixed to the boat and the other to the bank. If the force

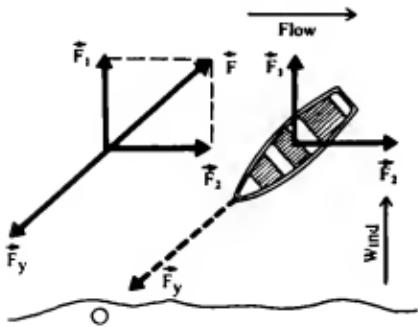


Fig. 147



Fig. 148

exerted by flowing water is, say, 150 N and the force of pressure of the wind is 100 N, the resultant of these two perpendicular forces can be calculated by the Pythagoras theorem:

$$F = \sqrt{F_1^2 + F_2^2}, F = \sqrt{(100 \text{ N})^2 + (150 \text{ N})^2} \simeq 180 \text{ N}.$$

Consequently, the boat can be held by the rope capable of withstanding tension not less than 180 N.

#### Exercise 24

1. A load is moved over a horizontal plane at a constant velocity with the help of two ropes to which forces of 500 N each are applied. The ropes form an angle of  $60^\circ$ . What is the friction on the plane? What is the magnitude of friction that requires the angle between the ropes to be  $0, 90, 120^\circ$ ?
2. A 3.0-kg sphere is hanging on a rope fixed to a smooth wall (Fig. 148). Find the tension of the rope and the force of pressure of the sphere on the wall. The rope forms an angle of  $15^\circ$  and passes through the centre of the sphere.
3. A 3.4-kg lamp is suspended at the middle of a 20-m horizontal rope, as a result of which the latter sags by 5 cm. Find the elastic forces emerging in the rope.

## 7.2.

### Equilibrium of Bodies with a Fixed Axis of Rotation

In the preceding section, we established the conditions of equilibrium for a body in the absence of rotation. But how can we ensure the absence of rotation?

To answer this question, let us consider a body that cannot translate but can turn or rotate. In order to make translation impossible, it is sufficient to fix the body at one point. For example, we can fix a board on a wall by nailing it up with a single nail. The translatory motion of such a board becomes impossible, but the board can turn with the nail as an axis.

**WHICH FORCES CAN CAUSE A BODY TO TURN?** Let us first find out which forces can and which cannot cause a turning (rotation) of a body having a fixed axis.

Figure 149 shows a body that can turn about an axis  $O$  normal to the plane of the figure. It can be seen that the forces  $\vec{F}_1$ ,  $\vec{F}_2$  and  $\vec{F}_3$  do not cause rotation of the body since their lines pass through the axis of rotation. Any force of this type is balanced by the reaction of the fixed axis.



Fig. 149

(a)



(b)



Fig. 150

Fig. 151



A turning (or rotation) can be caused by forces whose lines *do not pass through the axis of rotation*. For example, the force  $\bar{F}_1$  applied to the body as shown in Fig. 150a makes the body turn *clockwise*. On the other hand, the force  $\bar{F}_2$  (Fig. 150b) also causes the turning of the body but *anticlockwise*.

To make turning (rotation) impossible, we must obviously apply to the body at least two forces, one of which causes a clockwise rotation of the body, while the other rotates it anticlockwise. These forces may be not equal in magnitude to each other. For example, the force  $\bar{F}_1$  (Fig. 151) causes anticlockwise rotation of the body. It can be shown experimentally that it can be balanced by a force  $\bar{F}_2$ , rotating the body clockwise, whose magnitude is smaller than that of  $\bar{F}_1$ . This means that these two forces differing in magnitude produce the same "rotating action". What do they have in common? Which property is the same for them? Experiments show that in this case the product of the magnitude of the force and the distance<sup>11</sup> from the rotational axis is the same. This distance (denoted in Fig. 151 by  $d_1$  and  $d_2$  respectively) is called the *arm of force*. The arm of the force  $\bar{F}_1$  is  $d_2$  and the arm of the force  $\bar{F}_2$  is  $d_1$ .

**MOMENT OF FORCE.** Thus, the "rotating action" of a force is characterized by the product of its magnitude and the arm. *The quantity equal to the product of the magnitude of the force  $\bar{F}_1$  and its arm  $d$  is called the torque or the moment of force about the axis of rotation*

$$\underline{M = Fd}.$$

The expression "about the axis of rotation" is necessary in the definition of the torque since if we shift the axis of rotation from point  $O$  (see Fig. 151) to some other point without changing the magnitude of the force or its direction, then the arm of force, and hence its moment, will change.

The moment of force depends on two quantities: the magnitude of the

<sup>11</sup> The word "distance" here indicates the length of the perpendicular dropped from the axis of rotation onto the line of action of the force.

force and its arm. The same torque can be produced by a small force whose arm is large and by a large force with a small arm. If somebody, for example, is trying to shut the door by pushing it near the hinge, he could be thwarted by a child who is clever enough to push the door in the opposite direction by applying the force closer to the door edge. The door will remain at rest (Fig. 152).

Naturally, we must find a unit of measurement for the new quantity – the moment of force.

It follows from the expression  $M = Fd$  that for the unit of torque in SI should be taken the moment of force of 1 N whose line of action is separated from the axis of rotation by 1 m. This unit is called a newton-metre (N·m).

**CONDITION OF EQUILIBRIUM (RULE OF MOMENTS).** Moments of forces rotating a body anticlockwise are conventionally given the plus sign, while the minus sign corresponds to the torques rotating a body clockwise. Then the moments of forces  $\vec{F}_1$  and  $\vec{F}_2$  about the axis  $O$  (see Fig. 151) have opposite signs and their algebraic sum is zero. Thus, we can write the following condition of equilibrium for a body with a fixed axis:

$$F_1d_1 = F_2d_2 \quad \text{or} \quad -F_1d_1 + F_2d_2 = 0.$$

*A body that can rotate about a fixed axis is in equilibrium if the algebraic sum of the moments of forces applied to it about this axis is zero.*

This is the *rule of moments*, which is the condition of equilibrium for a body with a fixed axis of rotation.

We obtained the rule of moments for the case when two forces act on a body. It can be shown that this rule is valid also when several forces act on a body.

For this purpose, let us consider an experiment carried out with the help of the device shown in Fig. 153. It is a body of an irregular shape, fixed at an axis (rotational axis).

Forces are applied at four points of this body. Two of them are equal in magnitude to the weights of the corresponding loads shown in Fig. 153. The other two are elastic forces with which the stretched springs of the



Fig. 152



Fig. 153

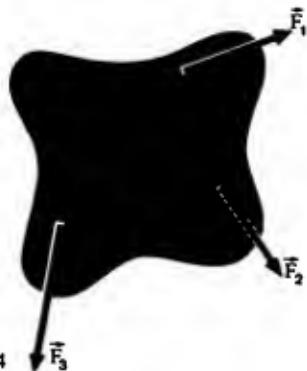


Fig. 154

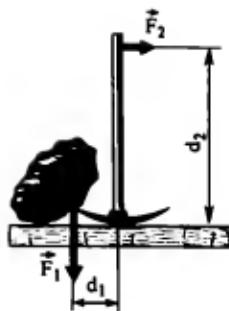


Fig. 155

dynamometers act on the body. The magnitudes of these forces are indicated on the scales of the dynamometers. Under the action of these four forces, the body is in equilibrium. Using a compass and a ruler, we can measure the arms of these forces. In doing so we can make sure that the algebraic sum of the moments of the four forces about the rotational axis is equal to zero.

Figure 154 schematically illustrates a similar experiment with three forces ( $\vec{F}_1$ ,  $\vec{F}_2$  and  $\vec{F}_3$ ) acting on a body. The fixed axis passes through point  $O$ . The figure shows that the moments of the forces  $\vec{F}_1$  and  $\vec{F}_2$  about the rotational axis of the body are negative, while the moment of the force  $\vec{F}_3$  is positive.

The condition of equilibrium for the body is written in the form

$$-F_1d_1 - F_2d_2 + F_3d_3 = 0,$$

where  $d_1$ ,  $d_2$  and  $d_3$  are the arms of the corresponding forces.

Let us now formulate the general condition of equilibrium for a body.

*For a body to be in equilibrium, it is necessary that the geometrical sum of the forces applied to the body and the sum of the moments of these forces about the rotational axis be equal to zero*<sup>11</sup>.

**LEVER RULE.** It can be easily seen that the rule of moments leads to the famous *lever rule*: *a lever is in equilibrium when the magnitudes of forces acting on it are inversely proportional to their arms.* This is just another expression for the rule of moments. Indeed, it follows from the formula  $F_1d_1 = F_2d_2$  that

$$\frac{F_1}{F_2} = \frac{d_2}{d_1}.$$

Figure 155 represents, by way of an example, a lever with two mutually perpendicular forces applied to it,  $\vec{F}_1$  and  $\vec{F}_2$ .

<sup>11</sup> The fulfilment of these conditions, however, does not prevent the body from moving uniformly in a straight line or from rotating at a constant period of revolution.

9

- Under which conditions does a force applied to a body cause its rotation about a fixed axis?
- What is called the arm of a force?
- Give the definition of the moment of force (torque).
- In which case is the torque acting on a pedal (Fig. 156a and b) larger?
- Formulate the condition of equilibrium for a body that can rotate about a fixed axis.
- Under which condition is the lever shown in Fig. 155 in equilibrium?

### EXAMPLE OF SOLVING A PROBLEM

A homogeneous rod of mass  $m = 2 \text{ kg}$  is fixed to a hinge at its lower end (Fig. 157). The rod is maintained in equilibrium by a horizontal guide rope fixed to a stationary vertical post. Using the data indicated on the figure, find the tension of the rope and the reaction of the hinge.

*Solution.* Three forces act on the rod: the force of gravity  $\vec{F}_g = m\vec{g}$ , applied at its middle, the elastic force  $\vec{F}_1$  of the rope and the elastic force  $\vec{F}_2$  of the hinge. The axis of rotation passes through the hinge at the lower end of the rod. Only two of the above forces have torques about this axis. The line of action of the normal reaction of the hinge passes through its axis; hence its moment is zero. Of the two remaining forces, the elastic force of the rope rotates the rod anticlockwise, while the force of gravity rotates it clockwise. According to the rule of moments, we have

$$-F_g d + F_1 d_1 = 0, \quad -2 \text{ kg} \times 10 \frac{\text{m}}{\text{s}^2} \times 0.30 \text{ m} + F_1 \times 0.40 \text{ m} = 0.$$

Solving this equation, we obtain  $F_1 = 15 \text{ N}$ .

In order to determine the normal reaction in the hinge, we apply the other condition of equilibrium: the sum of the projections of the applied forces onto each coordinate axis should be zero:

$$F_{2x} + F_{1x} = 0, \quad F_{2y} + F_{gy} = 0.$$



Fig. 156

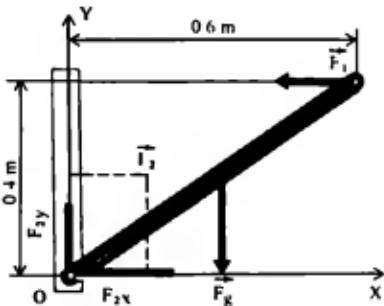
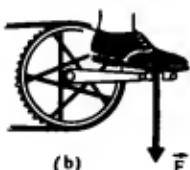


Fig. 157

Since

$$F_{1x} = -F_1 \quad \text{and} \quad F_{gj} = mg_y = -mg,$$

we have

$$F_{2x} - F_1 = 0, \quad F_{2x} = F_1, \quad F_{2x} = 15 \text{ N},$$

$$F_{2j} - mg = 0, \quad F_{2j} = mg, \quad F_{2j} = 20 \text{ N}.$$

By the Pythagoras theorem, we obtain

$$F_2 = \sqrt{(F_{2x})^2 + (F_{2j})^2},$$

$$F_2 = \sqrt{15^2 + 20^2} \text{ N} = 25 \text{ N}.$$

It follows from the conditions of equilibrium that the direction of the force  $\bar{F}_2$  should intersect the lines of action of the forces  $\bar{F}_1$  and  $\bar{F}_g$  at the same point.

### Exercise 25

- Figure 158 shows a homogeneous rod whose rotational axis passes through point  $O$ . Loads whose masses are 0.2 and 0.4 kg are suspended from points  $A$  and  $B$  respectively. What is the mass of the load which should be suspended from point  $C$  so that the rod is in equilibrium?
- A 0.8-kg load is fixed at point  $A$  of a homogeneous rod (Fig. 159) that can rotate about an axis. What should be the mass of the load fixed at point  $B$  so that the rod is in equilibrium? The mass of the rod is 400 g.
- A box is lying on an inclined plane. Will it slide down if the coefficient of friction between the box and the inclined plane is 0.2? The length of the inclined plane is 6 m and its height is 2 m.
- An aerial mast (Fig. 160) is fixed with the help of the guiding rope  $AB$  forming an angle of  $30^\circ$  with the mast. The force with which the aerial acts on the mast at point  $B$  (tension of the aerial) is 1000 N. Find the elastic force in the compressed mast and the force acting on the rope.



Fig. 158

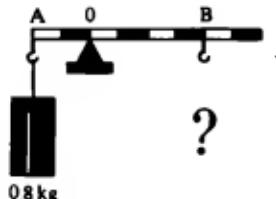


Fig. 159

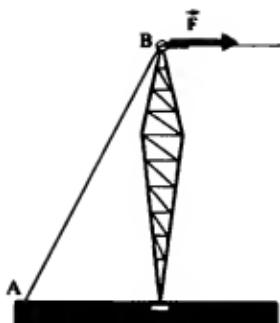


Fig. 160

#### Hometask

Give examples illustrating the practical use of a lever.

### 7.3. Stability of Equilibrium of Bodies

If a body is in equilibrium, this means that the sum of forces applied to it is equal to zero and the sum of the moments of these forces about the rotational axis is also zero. However, the question that arises now is whether the equilibrium is stable.

For example, it can be immediately seen that the equilibrium of a ball on the top of a convex support (Fig. 161) is unstable; a slight deviation of the ball from its equilibrium position leads to its rolling down. If, however, the same ball is placed on a concave support (Fig. 162), it is not easy to make it leave its place. In this case, the equilibrium of the ball can be assumed stable.

**WHAT IS THE SECRET OF STABILITY?** In the examples considered above, the ball is in equilibrium: its force of gravity  $F_g$  is equal and opposite to the elastic force (normal reaction)  $\bar{N}$  of the support (Figs. 163 and 164).

As a matter of fact, the slight deviation mentioned above plays a very important role here. Upon a very small deviation, which always occurs due to random vibrations, air flows and other reasons, the equilibrium of the ball is violated. It can be seen from Fig. 163 that as soon as the ball on a convex support leaves its initial position, the force of gravity  $F_g$  is no longer



Fig. 161



Fig. 162

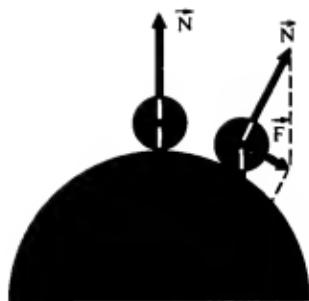


Fig. 163

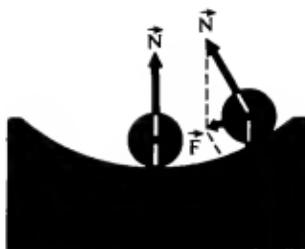


Fig. 164

balanced by the force  $\vec{N}$  exerted by the support (the force  $\vec{N}$  always acts normally to the surface of contact between the ball and the support). The resultant of the force of gravity  $\vec{F}_g$  and the normal reaction  $\vec{N}$ , viz. the force  $\vec{F}$ , is directed so that the ball moves still further from the equilibrium position.

Quite a different situation is observed if we consider a concave support (Fig. 164). Upon a small deviation from the initial position, equilibrium is violated in this case also. The elastic force exerted by the support does not balance the force of gravity in this case also. But the resultant  $\vec{F}$  is now directed so that the body returns to its initial position. This is the condition of stable equilibrium.

*Equilibrium of a body is stable if as a result of a small deviation from an equilibrium position the resultant of the forces applied to the body returns it to the equilibrium position.*

*Equilibrium is unstable if as a result of a small deviation of the body from an equilibrium position the resultant of forces applied to the body moves it away from this position.*

This is also valid for a body with a rotational axis. By way of an example, let us consider an ordinary ruler fixed on a rod passing through the hole near one of its ends (Fig. 165a, b). It can be seen from the figures that the position of the rule in Fig. 165a is stable. On the other hand, it is almost impossible to suspend the same ruler as shown in Fig. 166a. Any deviation from the vertical position (Fig. 166b) makes the ruler turn so that it occupies the position shown in Fig. 166c. Consequently, the equilibrium of the ruler corresponding to Fig. 166a is unstable.

Stable and unstable equilibrium also differ in the positions of the centre of gravity of the body. When the ball is in an unstable equilibrium (see Fig. 161), its centre of gravity is higher than in any neighbouring position. On the contrary, the centre of gravity of the ball in a stable equilibrium on the concave support (see Fig. 162) is lower than in any other neighbouring position. Consequently, *for a body to be in a stable equilibrium, its centre of gravity must be at the lowest possible position*

*The equilibrium of a body having a rotational axis is stable if its centre of gravity is below the axis of rotation.*

There exists one more type of equilibrium when a deviation from the equilibrium position does not lead to any change in the state of the body. Such is, for example, the position of a ball on a plane support (Fig. 167) or the position of a ruler suspended from a rod passing through a hole at its centre of gravity (Fig. 168). Obviously, upon any change in the position of the body it remains in equilibrium. Such equilibrium is called *neutral*.

**EQUILIBRIUM OF BODIES ON SUPPORTS.** We considered above the conditions of stable and unstable equilibrium for bodies having a fulcrum or rotational axis. The case when the contact between a support and a body is not a point (axis) but a certain surface is also very important for practical applications. A box on the floor, a glass on the table, various buildings and

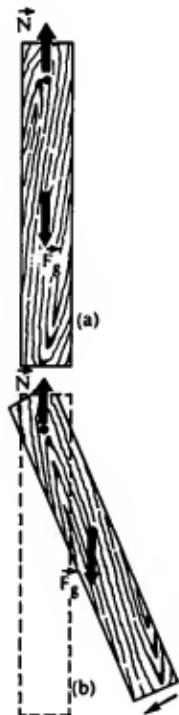


Fig. 165

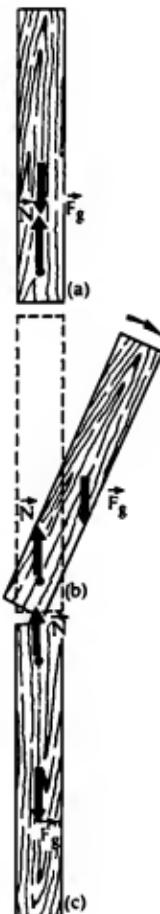


Fig. 166

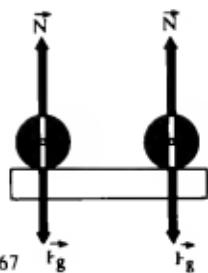


Fig. 167

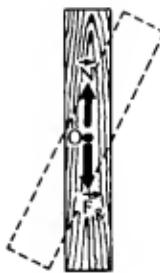


Fig. 168

chimneys have supporting surfaces. What are the conditions for stable equilibrium of bodies in this case?

As before, the bodies with supporting surfaces experience the action of two forces balancing each other, i.e. the force of gravity that can be assumed to be applied at the centre of gravity, and the elastic force (normal reaction) of the support normal to its surface. Just as in the cases considered above, an equilibrium is stable if a deviation from the equilibrium position does not give rise to a force moving the body away from this position. For example, when a prism rests on a horizontal surface (Fig. 169), it is naturally in equilibrium. This equilibrium is stable since upon a deviation by a small angle, the line of the force of gravity of the prism (which coincides with the vertical) intersects the base of the prism to the left of the supporting points (Fig. 170), and the force of gravity returns the prism to its original position.

If, however, the prism is inclined by a larger angle (Fig. 171), the result will be different. The line of the force of gravity (the vertical) now intersects the base of the prism on the right of the supporting points, and under the action of this force the prism is inclined still further. Ultimately, it falls.



Fig. 169

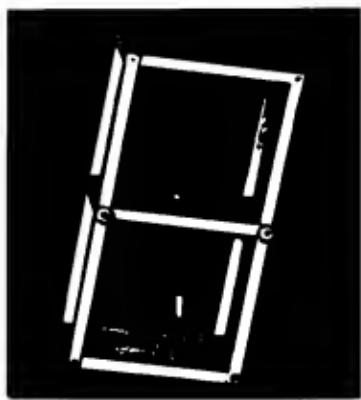


Fig. 170



Fig. 171



Fig. 172

Figure 172 shows the limiting position of the prism before it falls down. In this case, the line of action of the force of gravity crosses the line containing the supporting points of the prism.

Thus, for a body to be stable it is necessary that the vertical passing through the centre of gravity of the body intersects the supporting surface.

Supporting surface which determines the equilibrium is not always the surface that actually contacts the body. For example, a table contacts the floor only through its legs. However, the supporting surface of the table is the surface bounded by a contour which is formed by the straight lines connecting the legs. The supporting surface of a tripod (Fig. 173) is the triangle formed by the segments connecting its legs, etc.



Fig. 173

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1. Determine the type of equilibrium for the following cases:  
(a) a gymnast is hand-standing on parallel bars; a gymnast is hanging on rings, (b) a rope-walker is on a rope;  
(c) a wheel is put on an axle; (d) a ball is suspended from a thread; and (e) a ball is lying on a table.
2. In what way is a high stability of the following objects ensured: (a) a laboratory holder; (b) a tower crane;  
(c) a table lamp?
3. A lorry transported the following loads of the same mass: steel sheets, cotton and wood. In which case was the lorry more stable?

---

## Summary

The question of equilibrium of a body acted upon by forces is important in designing constructions that must be permanently at rest.

There are two conditions necessary for equilibrium:

- (1) the geometrical sum of the forces applied to a body must be zero;
- (2) the algebraic sum of the moments of applied forces about the rotational axis must be zero.

The moment of force about an axis is the quantity characterizing the rotational action of the force about this axis. It is equal to the product of the magnitude of force and its arm.

Not all types of equilibrium of a body can be realized in practice. Only stable and neutral equilibrium can be attained.

An equilibrium of a body is stable when upon a small deviation of the body from the equilibrium position, forces acting on the body return it to the original position.

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# Conservation Laws in Mechanics

## 8

### THE LAW OF CONSERVATION OF MOMENTUM

#### PHYSICAL QUANTITIES THAT ARE CONSERVED

In the previous chapters it was demonstrated how Newton's laws are used for solving problems on motion of bodies. It may seem that the study of mechanics could be finished here. However, in many cases it is very difficult to find the magnitudes of forces acting on a body. When we consider the collision of two bodies, say, two railway carriages, we know that they interact through elastic force. But it is very difficult, and sometimes even impossible to determine the magnitude of this force due to the complexity of deformations of the parts of the carriages in contact. Even in a simple case of collision between two balls, the form of deformation of each ball is very complex, and the values of quantities  $x$  and  $k$  which appear in Hooke's law are unknown:

$$(F_{\text{el}})_x = -kx.$$

In such cases, problems in mechanics are solved with the help of simple corollaries of the laws of motion, which are modifications of Newton's second law. These expressions involve, instead of forces and accelerations, new quantities. These quantities are *momentum* and *energy*. This part of the book is devoted to them. Momentum and energy are peculiar quantities possessing the property of *being conserved*. These quantities and their conservation play an important role not only in mechanics but also in other branches of physics. This explains a special attention paid to this quantities.

#### 8.1. Force and Momentum

##### Formula

$$\bar{F} = m\bar{a}, \quad (8.1.1)$$

which expresses Newton's second law, can be written in a different form if we recall that acceleration is the rate of variation of the velocity of a body. In

particular, for a uniformly accelerated motion, we have

$$\bar{a} = \frac{\bar{v} - \bar{v}_0}{t}. \quad (8.1.2)$$

Substituting this expression into (8.1), we obtain

$$\bar{F} = \frac{m(\bar{v} - \bar{v}_0)}{t}$$

or

$$\bar{F} = \frac{m\bar{v} - m\bar{v}_0}{t}. \quad (8.1.3)$$

This formula can also be written as

$$\bar{F}t = m\bar{v} - m\bar{v}_0. \quad (8.1.4)$$

Formula (8.1.4) is another expression of Newton's second law.

The right-hand side of this equation is the change in the product of the mass of a body and its velocity. This product is a physical quantity called the *momentum of a body*.

*The momentum of a body is the product of the mass of the body and its velocity.*

Momentum is a vector quantity. The direction of the momentum vector coincides with the direction of the velocity vector.

It is usually said that a body of mass  $m$  moving at a velocity  $\bar{v}$  carries (or has) the momentum  $m\bar{v}$ .

Obviously, the SI unit of momentum is the momentum of a body having a mass of 1 kg and moving at a velocity of 1 m/s. The unit of momentum is kilogram-metre per second ( $\text{kg} \cdot \text{m/s}$ ).

It can be seen from (8.1.4) that the change in momentum is equal to the product of the force  $\bar{F}$  and the time  $t$  of its action. The quantity  $\bar{F}t$  also has a special name. It is called the *impulse*.

*The change in the momentum of a body is equal to the impulse.*

When deriving formula (8.1.4), we assumed that the acceleration of the body, and hence the force acting on it, are constant. If, however, the force varies with time, the time interval during which this force is acting can be divided into small intervals during which the force can be considered constant. The change in momentum during each of small intervals can be calculated by (8.1.4). Summing up the obtained increments of momentum, we obtain the change in momentum over the entire time interval during which this force is acting.

If the time of action of a force is very short as, for example, during a collision of bodies or during an impact, we can use formula (8.1.4) directly, assuming that  $\bar{F}$  is the average force acting on the body.

A remarkable property of momentum is that under the action of a given force, it changes by the same value for all bodies if only the time of action of the force is the same. The same force acting during a certain time supplies the same momentum to a heavy barge as well as to a light boat.

---

1. What is the momentum of a body? What are the magnitude and direction of the momentum of a body?
2. What is the relation between the force acting on a body and its momentum? Can we say that a body has a momentum because a force is acting on it?
3. What is impulse? What are its magnitude and direction?
4. What is the relation between the impulse and the momentum of a body?
5. Which units are used for measuring the impulse and momentum of a body? Are they different?

---

### Exercise 26

1. Find the momentum of a 5-kg body and moving at a velocity of 2 m/s.
2. The 4-t tank of a street-sprinkling vehicle contains  $2\text{ m}^3$  of water. What is the momentum of the vehicle (a) when it moves at a velocity of 18 km/hr loaded with water; (b) when it moves at a velocity of 54 km/hr after all the water has run out?
3. A 20-g metallic ball falls at a velocity of 5 m/s on a steel slab, collides with it (elastic collision) and rebounds vertically upwards at the same initial velocity. Find the change in the momentum of the ball and the average force inducing the change, if the collision lasts for 0.1 s.
4. A driver puts the engine of his motorcar out of gear at a velocity of 72 km/hr. In 3.4 s the car stops. The friction of the wheels on the asphalt surface is 5880 N. What was the momentum of the car when the engine was put out of gear? What is the mass of the car?
5. A 2-t motorcar is moving at a velocity of 36 km/hr. What time is required for the car to come to a halt after the engine is put out of gear, if the friction of the wheels on the road is 5880 N?

---

### Hometask

Analyze the solutions of Problems 4 and 5 of Ex. 26 and find out which quantity determines the braking time of a moving body for a given magnitude of the decelerating force. Compare the result of the analysis with the formula given in Sec. 6.7.

---

## 8.2. The Law of Conservation of Momentum

Momentum possesses an interesting and important property inherent in quite a few physical quantities. This is the *property of being conserved*. It consists in that the geometrical sum of momenta of bodies which interact only with each other remains unchanged.

Of course, the momenta of the bodies change since forces of interaction act on each body, but the sum of momenta remains constant. This statement is called the *law of conservation of momentum*.

The law of conservation of momentum is one of the fundamental laws of nature. It can be proved quite easily when two bodies interact. Indeed, if the first body acts on the second one with the force  $\vec{F}$ , the second body acts on the first one with the force which, according to Newton's third law, is equal to  $-\vec{F}$ . Let us denote the masses of the bodies by  $m_1$  and  $m_2$  and their velocities relative to a reference system by  $\vec{v}_1$  and  $\vec{v}_2$ . As a result of interaction, the velocities of the bodies will change in a certain time  $t$  and become  $\vec{v}'_1$  and  $\vec{v}'_2$ .

According to (8.1.4), we have

$$\begin{aligned}\vec{F}t &= m_1 \vec{v}'_1 - m_1 \vec{v}_1, \\ -\vec{F}t &= m_2 \vec{v}'_2 - m_2 \vec{v}_2.\end{aligned}$$

Consequently,

$$m_1 \vec{v}'_1 - m_1 \vec{v}_1 = -(m_2 \vec{v}'_2 - m_2 \vec{v}_2).$$

Having reversed the signs on both sides of this equality, we write it as follows:

$$\boxed{m_1 \vec{v}_1 + m_2 \vec{v}_2 = m_1 \vec{v}'_1 + m_2 \vec{v}'_2.}$$

The left-hand side of this equation contains the sum of the initial momenta of the two bodies, while the right-hand side is the sum of the momenta of the same bodies in time  $t$ . These sums are equal. Thus, although the momentum of each body changes as a result of interaction, their total momentum, i.e. the sum of the momenta of the two bodies, remains unchanged, Q. E. D.

**WHEN IS THE LAW OF CONSERVATION OF MOMENTUM VALID?** It can be proved (and confirmed in experiments) that if many bodies interact, the geometrical sum of the momenta of these bodies or, as it is said, of a system of bodies, remains unchanged. It is only important that these bodies should interact only with one another and that other bodies, which are not involved in the system, should not act on them (or that these external forces should be balanced). Such a group of bodies which do not interact with other bodies outside this group is called a *closed system*. While we were considering the interaction of two bodies, we also assumed that other bodies are not acting on them. The *law of conservation of momentum* is valid just for closed systems.

The geometrical sum of the momenta of bodies comprising a closed system remains constant upon any interactions among the bodies of this system.

Hence it follows that an interaction among bodies consists in that some bodies impart a fraction of their momentum to others.

Momentum is a vector quantity. Therefore, if the sum of the momenta of bodies remains constant, the sum of projections of these momenta onto the coordinate axes also is conserved. For this reason, geometrical summation of momenta can be replaced by algebraic summation of their projections.

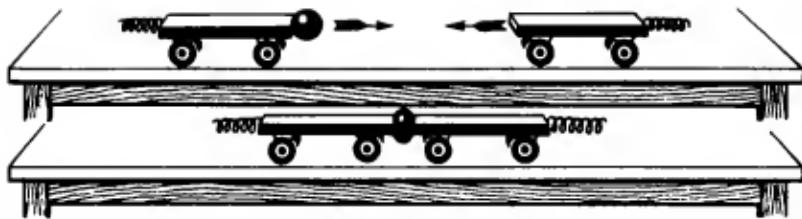


Fig. 174

The law of conservation of momentum can be illustrated by the following simple experiments.

1. Let us place two cars of the same mass  $m$  on rails. We attach to the end face of one of them a plasticine ball. Suppose that the cars move towards each other with the same velocity  $v$  (Fig. 174). The cars stop when they meet. The results of the experiment can be easily explained. Before meeting, the momentum of the left car is  $mv$  and that of the right car is  $-mv$  (the cars were moving with opposite velocities). Therefore, the total momentum of the cars before meeting was zero:

$$mv + (-mv) = 0.$$

After the collision, the cars stopped. Consequently, the total momentum of the cars remains zero.

2. We can turn the cars so that they are facing each other with spring buffers (Fig. 175). Repeating the experiment, we see that after the collision the cars will move apart. In this interaction, the velocities of the cars reverse their directions while their magnitudes remain the same as before the interaction. If before meeting the momenta of the left and right cars were  $mv$  and  $-mv$  respectively, after their meeting the momenta will be  $-mv$  and  $mv$  respectively. Therefore, the total momentum of the two cars is zero both before and after the collision, in correspondence with the law of conservation of momentum.

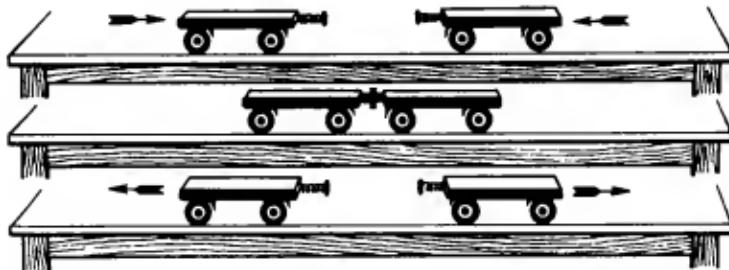


Fig. 175

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1. Formulate the law of conservation of momentum.
2. What is called a closed system of bodies?
3. A sailing boat has got into calm and came to a halt. Is it possible to make it move by blowing up its sails with the help of a pump or bellows mounted on board?
4. A gun shot was made from a moving tank. Does the shot affect the velocity of the tank? Which bodies form a closed system in this case?
5. Two balls of equal mass are rolling towards each other over a very smooth surface at velocities of equal magnitude (hence the balls form a closed system). After collision they move apart at the same velocities (in magnitude). What is their total momentum before collision, at the moment of collision and after it?
6. Can the fragments of an exploding grenade move in the same direction if before the explosion the grenade was at rest? And if it was moving?

#### EXAMPLE OF SOLVING A PROBLEM

A railway carriage whose mass is 30 000 kg is moving at a velocity of 1.5 m/s. It is clutched with a stationary carriage whose mass is 20 000 kg. What is the velocity of the cars after clutching if the railroad segment where this process occurs is straight?

*Solution.* Let us direct the coordinate axis along the velocity vector of the first carriage. We denote the mass of the first (moving) carriage by  $m_1$ , the mass of the second (stationary) carriage by  $m_2$ , the velocity of the first carriage before clutching by  $v_1$  and the common velocity of the two carriages after clutching by  $v$ . According to the law of conservation of momentum, the total momentum of the two carriages before and after the clutch must be the same.

Before the clutch, the projection of the total momentum onto the coordinate axis is positive and equal to the magnitude of the momentum of the first car, viz.  $m_1 v_1$ . After the clutch, the projection of the total momentum should remain positive and equal to the magnitude of the momentum vector of clutched cars, viz.  $(m_1 + m_2) v$ .

In accordance with the law of conservation of momentum, we write

$$m_1 v_1 = (m_1 + m_2) v.$$

Hence

$$v = \frac{m_1}{m_1 + m_2} v_1,$$

$$v = \frac{3 \times 10^4 \text{ kg} \times 1.5 \text{ m/s}}{5 \times 10^4 \text{ kg}} = 0.9 \text{ m/s.}$$

In this problem, we have not calculated the magnitudes of the forces of interaction between the carriages. However, using the momentum conservation law, we determined the velocities of the bodies we are interested in. Obviously, if the initial positions of the bodies are known, we can determine the positions of these bodies at any instant of time from their velocities. For this reason, the law of conservation of momentum is very important. It allows us to solve the fundamental problem of mechanics.

---

### Exercise 27

1. A man whose mass is 70 kg, running at a velocity of 7 m/s, catches up with a car which has a mass of 30 kg and a velocity of 2 m/s and jumps onto it. What is the velocity of the car after that?
2. In the formation of a train, three clutched wagons moving at a velocity of 0.4 m/s collide with a stationary wagon. After this the four wagons start moving in the same direction with the same velocity. Calculate the velocity if the masses of the wagons are equal.
3. An anti-aircraft shell shot in the vertical direction exploded after having reached its maximum height and splitted into three fragments. Two of them scattered at right angles, their masses being 9 and 18 kg and the velocities 60 and 40 m/s respectively. The third fragment had the velocity of 200 m/s. Find graphically the direction of flight of the third fragment. What is its mass?

---

## 8.3. Reaction Propulsion

*Reaction propulsion* is an interesting and important case of practical application of the law of conservation of momentum. This term is applied to the motion of a body due to the separation of its certain part with some velocity.

For example, the motion of *rockets* is a reaction propulsion. Any rocket is a two-body system. It consists of a shell and a fuel contained in it. The shell is made in the form of a tube with one end closed and the other open and supplied with a fitting having an opening of a special shape, called the propelling nozzle.

During the firing of a rocket, the fuel is burnt and converted into a high-pressure gas having a very high temperature. Due to high pressure, this gas escapes from the nozzle at a high velocity. The rocket flies in the opposite direction (Fig. 176).

Before launching, the total momentum of the rocket (shell and fuel) in the coordinate system fixed to the Earth is zero. As a result of interaction of the gas and the shell, the ejected gas acquires a certain momentum. We assume that the effect of the force of gravity is negligibly small. Then the shell and



Fig. 176

fuel can be considered as a closed system, and their total momentum must also be zero after launching. Consequently, due to the interaction with the gas, the shell acquires a momentum equal in magnitude to the momentum of the gas and having the opposite direction. Hence, not only the gas but the rocket shell is also set in motion. It may contain instruments for scientific researches, communication equipment, etc. It may carry a spacecraft with cosmonauts on board.

The law of momentum conservation allows us to determine the velocity of the rocket (shell).

Indeed, let us first suppose that all the gas formed upon combustion of the fuel is ejected from the rocket at once and not flows out gradually.

We denote the entire mass of the gas obtained from the rocket fuel by  $m_g$  and its flow velocity by  $v_g$ . The mass and velocity of the shell are denoted by  $m_{sh}$  and  $v_{sh}$ . According to the law of momentum conservation, the sum of the momenta of the shell and gas after launching and before it must be the same, i.e. it must be equal to zero. Consequently,

$$m_g(v_g)_y + m_{sh}(v_{sh})_y = 0$$

or

$$m_{sh}v_{sh} = m_g v_g$$

(the  $Y$ -axis is chosen in the direction of motion of the shell). Hence we can find the shell velocity:

$$v_{sh} = \frac{m_g}{m_{sh}} v_g. \quad (8.3.1)$$

This formula shows that the velocity of the rocket shell is the higher, the larger the velocity of the ejected gas and the larger the ratio of the mass of the fuel to the mass of the shell. Therefore, the shell will acquire

Konstantin Eduardovich Tsiolkovsky (1857-1935) was the Russian scientist and inventor, the founder of cosmonautics. Since 1880th, he was engaged in designing airships, aeroplanes and rockets. Tsiolkovsky put forward the idea of using rockets in space flights. In this field, he obtained the results that remain important even today. His ideas about rockets, reaction propulsion engines and space flights had a strong effect on the development of rocketry and space engineering in the Soviet Union and abroad.



a considerable velocity if the mass of the fuel is much larger than the mass of the shell. For example, in order that the shell velocity be four times as large as the velocity of the ejected gas, the mass of the fuel must be four times larger than that of the shell, i.e. the shell mass must amount to one fifth of the mass of the rocket at the start. Unfortunately, the "useful" part of the rocket is just the shell.

We assumed that all the gas is ejected from the rocket at once. In actual practice, it flows out not immediately, although quite rapidly. This means that as the fuel is consumed and the rocket velocity increases, the velocity of the ejected gas relative to the Earth becomes smaller. Hence the momentum acquired by the rocket due to the gas ejection also decreases. As a result, the velocity  $v_{sh}$  of the rocket turns out to be lower than the one calculated from (8.3.1).

This circumstance explains why the mass of fuel required for attaining a given velocity should be considerably larger than it follows from our calculations. High-accuracy calculations show that for the shell velocity to be four times higher than the gas velocity, the mass of the fuel at the start must be not four but dozens of times larger than the mass of the shell. Moreover, if we take into account the drag force of the air through which the rocket must fly after being launched from the Earth and the attraction to the Earth, we may draw the conclusion that this ratio must be still higher.

In contrast to other transport facilities, the rocket can move without interacting with any other bodies besides the combustion products of the fuel contained in the rocket itself.

That is why rockets are used for launching artificial satellites of the Earth and spacecraft to the space. In the space, they have no support or anything from which they could push off like terrestrial transport facilities.

If necessary, a rocket can be decelerated. It is just what cosmonauts do at the end of a space flight, when they must reduce the velocity of their spacecraft to return to the Earth. It is clear that a rocket reduces its velocity if the gas is ejected from the nozzle in the direction of motion of the rocket.



Sergei Pavlovich Korolev (1907-1966)

The idea of using rockets for space flights was put forward as early as at the beginning of the 20th century by the famous Russian scientist K. E. TSIOLKOVSKY. This idea was realized by Soviet scientists and engineers headed by the remarkable Soviet scientist S. P. KOROLEV. Hundreds of artificial Earth's satellites and spacecraft are launched to space by rockets. Thanks to rockets, human beings visited the Moon. With the help of rockets, space laboratories were brought to the Moon and artificial Moon satellites were launched.

The first artificial satellite of the Earth was launched with the help of a rocket on October 4, 1957, in the Soviet Union.

The first man who appeared in the space on the artificial satellite of the Earth was the Soviet citizen YURI GAGARIN. On April 12, 1961, he orbited the Earth on the *Vostok* spacecraft.

Soviet rockets were the first to reach the Moon, fly it around and photograph its invisible "rear" side. It was the Soviet rocket that reached the Venus for the first time. The USSR occupies the leading position in the exploration of cosmic space.

?

1. It is known that a rocket can acquire an acceleration in space where there are no bodies around. On the other hand, an acceleration can be only imparted by a force, which is a result of action of one body on another. Then why is the rocket accelerated?
2. What determines the velocity of a rocket?
3. How can a spacecraft be decelerated?
4. Which of the three types of forces described in Chapter 5 does the force imparting an acceleration to a rocket belong to?



Yuri Alekseevich Gagarin (1934 -1968)

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## Summary

One of the most important characteristics of motion of a body is its momentum. It is a vector quantity defined as the product of the mass of a body and its velocity.

The same force  $\vec{F}$  acting during a certain time  $t$  imparts the same momentum to all bodies, which is equal to the impulse  $\vec{F}t$ .

Momentum obeys the *law of conservation*. The total momentum of bodies constituting a closed system remains unchanged upon any interactions and any motions of the bodies in this system.

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## ONE OF THE MOST IMPORTANT QUANTITIES IN SCIENCE AND ENGINEERING

It was shown in the previous chapter that the quantity (called momentum) for which the law of conservation is valid plays a very important role. There is another, not less important, quantity which also remains constant (is conserved) for a closed system. This is *energy* which is dealt with not only in mechanics but also in other branches of physics, as well as in natural sciences and engineering. However, before studying the concept of energy and the energy conservation law, we should get acquainted with the quantity called *mechanical work* (or simply *work*). This quantity is closely related to energy. It is also important in itself since the operation of various machines and mechanisms, as well as the working activity of people, is often reduced to mechanical work. What is work?

## 9.1. Mechanical Work

The quantity which is now called mechanical work appeared in mechanics only in the 19th century (almost 150 years after Newton has discovered his laws of motion), when power tools and engines were employed more widely. An engine in operation is said to be "working".

The concept of mechanical work has been introduced in the introductory course of physics (see *Junior Physics*, Sec. 59). It was found that *when a constant force  $F$  acts on a body and the body is displaced in the direction of force by  $s$ , the work done in this case is equal to the product of the magnitudes of the force and the displacement*:

$$A = Fs.$$

The unit of work, a joule (J), was also introduced. It should be recalled that *the unit of work in SI (joule) is the work done by a force of 1 N upon a displacement of the point of its application by 1 m*:

$$1 \text{ J} = 1 \text{ N} \cdot \text{m}.$$

**POSITIVE AND NEGATIVE WORK.** In the introductory course of physics, we considered the work done by a force whose direction coincides with the direction of motion of a body. In this case, the body moves with an acceleration. However, the body is often acted upon not by one but by several forces. How can the work of these forces be calculated?

Let us first consider the case when a body moves uniformly in a straight line. In this case, the vector sum of the forces acting on the body is zero. For

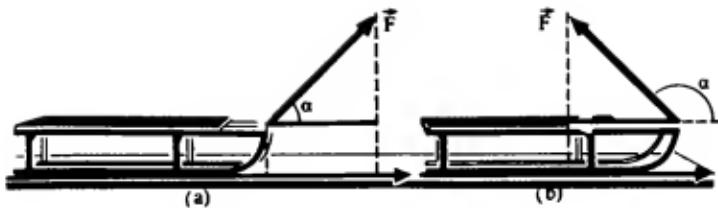


Fig. 177

example, when a load is being lifted uniformly by a crane, the load experiences the action of the tension of the rope, which is directed along the displacement (upwards), and the force of gravity which is directed against the motion (downwards). When a man moves a safe over the floor, it is acted upon by the muscular force, with which the man pulls or pushes it, and by the sliding friction directed oppositely to its motion.

Do forces directed oppositely to the displacement perform work? In order to answer this question, let us recall that if the vector sum of the forces applied to a body is zero, the situation should be the same as if no forces acted on the body. Then the total work of all the forces applied to the body should also be zero. For this, the work of some forces should be positive while the work of other forces should be negative. Otherwise, their sum could not be zero. A positive work is done by forces that have the same direction of the displacement, while forces directed oppositely to the displacement do a negative work.

**GENERAL EXPRESSION FOR THE WORK DONE BY A FORCE.** Forces applied to a moving body often form an angle with the direction of the displacement, which is neither  $0$  nor  $180^\circ$ . For example, a force applied to a sledge sliding along a horizontal road (Fig. 177a and b) forms an angle  $\alpha$  with the horizontal. In the first case (Fig. 177a), the angle  $\alpha$  is acute, while in the second case, it is obtuse (Fig. 177b). In order to calculate the work in all the cases, the formula for work should be written as

$$A = Fs \cos \alpha. \quad (9.1.1)$$

Here  $\alpha$  is the angle between the force and displacement vectors.

Indeed, if vectors  $\vec{F}$  and  $\vec{s}$  have the same direction, the angle  $\alpha$  between them is zero. We know that  $\cos 0^\circ = 1$ . In this case, the work  $A = Fs$ . If the vectors  $\vec{F}$  and  $\vec{s}$  have opposite directions,  $\alpha = 180^\circ$ ,  $\cos 180^\circ = -1$ , and the work  $A = -Fs$ . When the angle  $\alpha$  is acute (see Fig. 177a), its cosine is positive, and hence the work done by such a force is also positive. If, however, the angle  $\alpha$  is obtuse (Fig. 177b), its cosine is negative and the work done by such a force is also negative.

*The work done by a constant force is equal to the product of the magnitudes of the force and the displacement multiplied by the cosine of the angle formed by the vectors of force and displacement*

It should be emphasized that, as can be seen from Fig. 177a and b, the quantity  $F \cos \alpha$  is the projection of the force onto the direction of

displacement. Hence, the work done by a force can be defined as the product of the magnitude of the displacement of a body and the projection of the force onto the direction of displacement.

Formula (9.1.1) shows that work is a *scalar*, although both force and displacement are vectors. We cannot assign any direction to work.

**WHEN IS THE WORK DONE BY A FORCE EQUAL TO ZERO?** The direction of a force can be at right angles to the displacement of a body. In this case,  $\alpha = 90^\circ$ ,  $\cos \alpha = 0$  and  $A = 0$ . For example, when a load is moved in a horizontal direction, the force of gravity acting on it is normal to the direction of the displacement. Therefore, when a body moves over a horizontal plane, the work done by the force of gravity is zero. The force that makes the body move uniformly in a circle also does no work since, as is well known, it is directed along the radius towards the centre of the circle and thus is normal to the direction of displacement at any point. For example, the tension of a thread attached to a body moving uniformly in a circle does no work like the force of universal gravitation which makes artificial Earth's satellites move in circular orbits.

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1. When a force can be said to do work?
2. When a force does a positive work? When its work is negative?
3. Write the expression for the work done by a force directed at an angle to the displacement of the body.
4. Under which condition does a force applied to a moving body perform no work?
5. A motorcar moves along a smooth horizontal road. Does the force of gravity acting on the car perform any work?
6. Does the force of attraction of the Moon by the Earth perform any work in the orbital motion around the Earth?

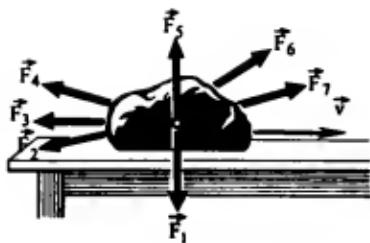


Fig. 178

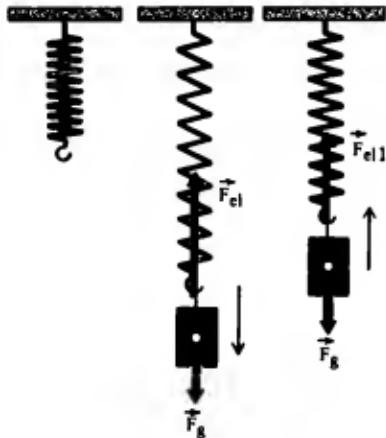


Fig. 179

(Assume that the orbit of the Moon is circular.)

7. Figure 178 shows a body to which several forces are applied. Which of them perform a positive work and perform a negative work?
8. A body is thrown vertically upwards. Does the force of gravity perform positive work or negative work when the body is (a) ascending; (b) falling down?

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### Exercise 28

1. A load sliding with friction over a flat horizontal surface is acted upon by a force of 200 N directed at  $60^\circ$  to the horizontal. What is the work done by this force in displacing the load by 5 m, if the motion occurs at a constant velocity? What is the coefficient of friction between the load and the surface of the plane, if the mass of the load is 31 kg?
2. A mountain skier whose mass is 70 kg is lifted by an elevator along a slope 180 m long, forming an angle of  $60^\circ$  with the horizontal. Calculate the work done by the force of gravity acting on the skier. What is the sign of this work? What is the work done by the elevator on the skier, if it is ascending at a constant velocity?

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### Hometask

Analyze Fig. 179 and find out (a) in which case the elastic force  $\vec{F}_{el}$  does a positive work and when its work is negative; (b) when the force of gravity  $\vec{F}_g$  does a positive work and when it does a negative work.

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## 9.2.

### Work Done by Forces Applied to a Body and the Change in Its Velocity

Let us consider a constant force  $\vec{F}$  acting on a body (it can be the resultant of several forces). It can be said about the force  $\vec{F}$  that, firstly, it imparts an acceleration to the body, due to which its velocity varies, and, secondly, the force  $\vec{F}$  does a work since the body is moving. We can expect that there is a certain relation between the work done by the force and the change in the velocity of the body. Let us establish this relation.

We consider a simple case when the force and displacement vectors are in one direction along the same straight line (Fig. 180). Let us choose the coordinate axis in the same direction. Then the projections of the force  $\vec{F}$ , displacement  $\vec{s}$ , acceleration  $\vec{a}$  and velocity  $\vec{v}$  are equal to the magnitudes of the vectors.

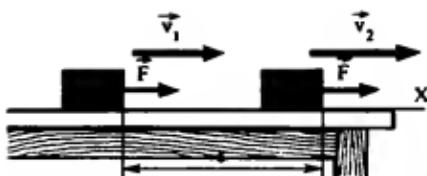


Fig. 180

The work done by the force in this case is

$$A = Fs. \quad (9.2.1)$$

According to Newton's second law, we have

$$F = ma. \quad (9.2.2)$$

It was shown in Chapter 2 that in a uniformly accelerated rectilinear motion, the velocity and displacement are connected through the following relation:

$$s = \frac{v_2^2 - v_1^2}{2a}, \quad (9.2.3)$$

where  $v_1$  and  $v_2$  are the magnitudes of the initial and final velocity vectors on the segment of the path under consideration.

Substituting into formula (9.2.1) expressions (9.2.2) and (9.2.3) for  $F$  and  $s$ , we obtain

$$A = Fs = ma \frac{v_2^2 - v_1^2}{2a} = \frac{mv_2^2}{2} - \frac{mv_1^2}{2}. \quad (9.2.4)$$

We obtained the formula relating the work done by the force  $F$  to the change in the velocity of the body (to be more precise, the square of its velocity).

**KINETIC ENERGY.** The expression on the right-hand side of formula (9.2.4) is the change in the quantity  $mv^2/2$ , i.e. half the product of the mass of a body and the square of its velocity.

This quantity is called the *kinetic energy of a body* and is denoted by  $E_k$ . Then formula (9.2.4) can be written as

$$\underline{A = E_{k2} - E_{k1}}. \quad (9.2.5)$$

The work done by the resultant of forces applied to a body is equal to the change in its kinetic energy.

This statement is called the *kinetic energy theorem*.

When a force acting on a body is directed along its trajectory of motion, and hence does a positive work, then  $mv_2^2/2 - mv_1^2/2 > 0$ . This means that  $mv_2^2/2 > mv_1^2/2$ , i.e. the kinetic energy of the body increases. This is what should be expected, since the force directed along the displacement of the body increases the magnitude of its velocity. It can be easily seen that when the force is directed against the displacement, and hence does a negative work, the kinetic energy of the body decreases.

It follows from formula (9.2.5) that kinetic energy is expressed in the same units as work, i.e. in joules.

We proved the theorem on kinetic energy with the help of Newton's second law. Therefore, it is valid irrespective of the kind of forces acting on a body: elastic forces, friction or the force of universal gravitation, and in particular, the force of gravity.

It can also be shown that the theorem on kinetic energy is valid as well when the force is not constant and when its direction does not coincide with that of the displacement.

The physical meaning of kinetic energy is easy to grasp.

Let us suppose that a body of mass  $m$  at rest ( $v_0 = 0$ ) has to be imparted a velocity  $v$ . For example, the velocity  $v$  should be imparted to a shell which is at rest in the barrel of a gun. A certain work  $A$  should be done for that. What is this work?

It follows from the theorem on kinetic energy that

$$A = \frac{mv^2}{2} - 0 = \frac{mv^2}{2}.$$

Consequently, the kinetic energy of a body of mass  $m$ , moving at a velocity  $v$ , is equal to the work that must be done by a force acting on the body at rest in order to impart this velocity to it. The work of the same magnitude is required to stop the body.

The kinetic energy theorem also implies that kinetic energy is a physical quantity characterizing a moving body. Its change is equal to the work done by the force acting on the body.

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1. Define the kinetic energy of a body. Is it a scalar or a vector?
2. Formulate the kinetic energy theorem.
3. How does the kinetic energy of a body vary if the force applied to it does a positive work?
4. What is the variation of the kinetic energy of a body if the force applied to it does a negative work?
5. Does the kinetic energy of a moving body change upon a variation of the direction of its velocity vector?
6. Two balls of the same mass roll towards each other with the same velocity over a very smooth surface. The balls collide and then move in the opposite directions with equal velocities. What is their total kinetic energy before collision, at the moment of the collision and after that?

#### EXAMPLE OF SOLVING A PROBLEM

What is the work that must be done to increase the velocity of a train from  $v_1 = 72$  km/hr to  $v_2 = 108$  km/hr, if the mass  $m$  of the train is 1000 t? What force must be applied to the train to attain this increase in velocity over a distance of 2000 m? Assume that the motion is uniformly accelerated.

*Solution.* The work  $A$  can be obtained from the formula

$$A = \frac{mv_2^2}{2} - \frac{mv_1^2}{2}.$$

Substituting into this formula the data of the problem, we get

$$A = \frac{10^6 \text{ kg} (30 \text{ m/s})^2}{2} - \frac{10^6 \text{ kg} (20 \text{ m/s})^2}{2} = 250 \times 10^6 \text{ J}$$
$$= 250000 \text{ kJ.}$$

By definition, the work  $A = Fs$ . Consequently,

$$F = \frac{A}{s}, \quad F = \frac{250 \times 10^6 \text{ J}}{2000 \text{ m}} = 125000 \text{ N} = 125 \text{ kN.}$$

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### Exercise 29

1. The force of 40 N is applied to a 3.0-kg body at rest. After this, the body covers a distance of 3.0 m on a smooth horizontal surface without friction. Then the force is reduced to 20 N, and the body covers another 3.0 m. Find the kinetic energy of the body and its velocity at the end of the second segment.
2. What is the work that must be done to stop a 1000-t train moving at a velocity of 108 km/hr?
3. Calculate the kinetic energy of an artificial satellite of the Earth having the mass of 1300 kg and moving in a circular orbit at a height of 100 km above the surface of the Earth.
4. A body having a kinetic energy of 10 J moves uniformly in a circle of radius 0.5 m. What is the force acting on the body? What is its direction? What is the work done by this force?
5. The driver of a motorcar put the engine out of gear at a velocity of 72 km/hr. Having passed 34.0 m, the car stopped. What was the kinetic energy of the car at the moment of disengagement, if the friction of the wheels on the surface of the road is 5880 N? What is the mass of the car?
6. A 4-t motorcar moves at a speed of 36 km/hr. What distance does it cover before coming to a halt, if the friction of the wheels on the surface of the road is 5880 N?

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### Hometask

Analyze solutions of Problems 5 and 6, Ex. 29, and find the quantity which determines the braking distance of a moving body for a given magnitude of the decelerating force. Compare the result of the analysis with the formula given in Sec. 6.7.

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## 9.3.

### Work Done by the Force of Gravity

It was mentioned above that the kinetic energy theorem is valid for any forces since this theorem is a direct consequence of Newton's second law. However, the work done by each of the mechanical forces known to us can be calculated using the formulas obtained for these forces in Chapter 5 instead of the kinetic energy theorem.

Let us start with the force of gravity, viz. the force with which the Earth acts on a body near its surface, where this force can be assumed constant and equal to  $m\bar{g}$  ( $m$  is the mass of the body and  $\bar{g}$  is the free-fall acceleration).

When a body moves vertically downwards the direction of the force of gravity coincides with that of the displacement. When the body moves from the height  $h_1$  above a certain level (taken as the reference level) to the height  $h_2$  above the same level (Fig. 181), the magnitude of its displacement is  $h_1 - h_2$ . Since the directions of the displacement and force coincide, the work done by the force of gravity is positive:

$$A = mg(h_1 - h_2). \quad (9.3.1)$$

The heights  $h_1$  and  $h_2$  need not be measured from the surface of the Earth. Any level can be taken as the reference level. It can be the floor of the room, a table or a chair, the bottom of a hole in the ground and so on, since the formula for work involves only the difference in elevations, which is independent of the reference level. It is only necessary to determine the elevation of the body for different positions relative to the same level. This level can be taken as the *zero level*.

For example, we could take the level  $B$  (Fig. 181) as the zero level. Then the work would be given by the following expression:

$$A = mgh, \quad (9.3.2)$$

where  $h$  is the vertical distance between the levels  $A$  and  $B$ .

If a body moves upwards, the force of gravity is directed against the displacement, and the work done by this force is negative. If a body rises from the zero level to a height  $h$ , the force of gravity does the work

$$A = -mgh.$$

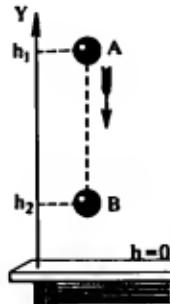


Fig. 181

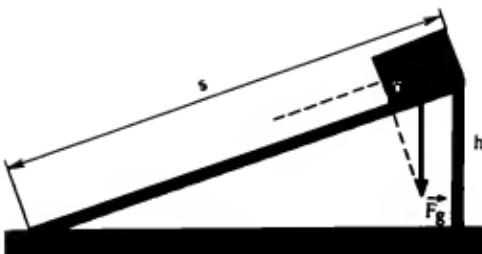


Fig. 182

**WHAT ARE INCLINED PLANES USED FOR?** Let us now find the work done by the force of gravity when a body does not move along the vertical.

By way of an example, let us consider the motion of a body along an inclined plane (Fig. 182). Suppose that a body of mass  $m$  on an inclined plane of height  $h$  is displaced by  $s$  whose magnitude is equal to the length of the inclined plane. The work done by the force of gravity  $\bar{F}_g = mg$  in this case should be calculated by the formula  $A = mgs \cos \alpha$ . It can be seen from the figure that

$$s \cos \alpha = h.$$

Hence

$$A = mgh.$$

This expression for work coincides with (9.3.2).

It turns out that the work done by the force of gravity does not depend on whether a body moves along the vertical or covers a longer distance along an inclined plane. For the same "loss of height"  $h$ , the work done by the force of gravity is the same (Fig. 183).

Why is an inclined plane often used in engineering and in everyday life for lifting loads although the work done to displace the load along the inclined plane is the same as the work done to lift it along the vertical?

While solving Problem 1 (p. 145, see Fig. 129), we found out that in the absence of friction, the acceleration of a body sliding along an inclined plane is equal in magnitude to  $g \sin \alpha$ , i.e. it is less than  $g$ . This means that the inclined plane as if reduced the force of gravity (yielding  $mg \sin \alpha$  instead of  $mg$ ). Of course, the force of attraction to the Earth actually is not decreased

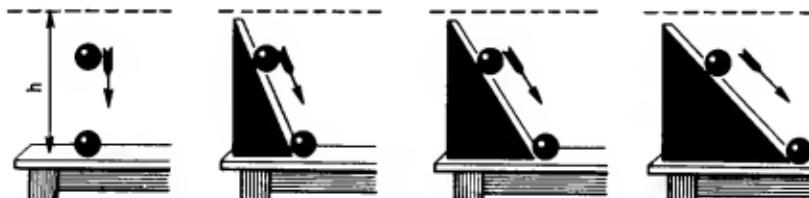


Fig. 183



Fig. 184

(a)



(b)

(it is always  $mg$ ). In a uniform motion along the inclined plane, the force applied to the body in the direction of the displacement is less than the force of gravity. True, the load in this case covers a larger distance. The longer distance is the price for the ability to lift a load by a smaller force using an inclined plane. In mountains, for example, the length of roads is artificially increased by building hairpin bends. This reduces the force required for a train or a motorcar to climb this road.

Inclined planes are used in "devices" like a plough (a ploughshare operates as an inclined plane), a wheelbarrow (a combination of an inclined plane and a lever), and so on.

Using inclined planes, we "gain" not only in the force applied to a body for lifting it to a certain height but also in other forces. For example, use is made of wedge, which is a combination of two inclined planes (Fig. 184a). A knife and other cutting tools are modifications of a wedge.

Figure 184b is a schematic diagram of forces acting on the blade of a wood-chopper. The force  $F_1$  acting on the wedge when a sledge-hammer strikes its rear part turns out to have much smaller magnitude than the normal reactions  $N_1$  and  $N_2$  of chopped wood. At a small wedge angle ( $2\alpha = 25^\circ$ ), the gain in force is about five-fold.

A helical gear is also an inclined plane wound over a rod (Fig. 185).

**PECULIAR FEATURE OF THE WORK DONE BY THE FORCE OF GRAVITY.** The work done by the force of gravity is determined by the "loss



Fig. 185

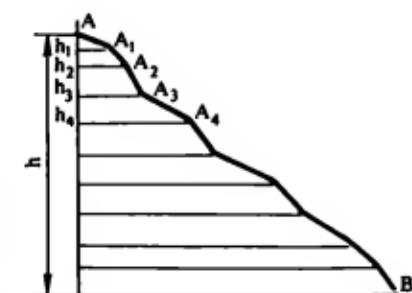


Fig. 186

in height" not only in the case of an inclined plane. This is true for the motion along any trajectory. Indeed, suppose that a body moves along an arbitrary path, like, for example, the one shown in Fig. 186. This trajectory can be divided into a number of small segments  $AA_1$ ,  $A_1A_2$ ,  $A_2A_3$ , etc. Each segment can be treated as a small inclined plane, and the entire motion of the body along the path  $AB$  can be represented as the motion along the set of inclined planes following one another. The work done by the force of gravity on each inclined plane is the product of  $mg$  and the change in the height of the body over this segment. If the changes in heights on individual segments are  $h_1$ ,  $h_2$ ,  $h_3$ , ..., then the magnitudes of the work done by the force of gravity over these segments are  $mgh_1$ ,  $mgh_2$ ,  $mgh_3$ , etc. The total work over the entire path can be found by summing up these works:

$$A = mgh_1 + mgh_2 + mgh_3 + \dots = mg(h_1 + h_2 + h_3 + \dots).$$

But

$$h_1 + h_2 + h_3 + \dots = h.$$

Consequently,

$$A = mgh.$$

Thus, the work done by the force of gravity does not depend on the path of a body and is always equal to the product of the magnitude of the force of gravity and the difference in heights of the initial and final positions. When a body moves downwards, the work is positive, while for the upward motion the work is negative.

If after rising to a certain height a body returns to the initial position, the work done for such a closed path is zero. This is a peculiar feature of the force of gravity: the work done by it over a closed path is equal to zero.

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1. Does the work done by the force of gravity depend on the length of the distance covered by a body and on its mass?
2. A body thrown at a certain angle to the horizontal describes a parabola and falls to the Earth. What is the work done by the force of gravity, if the initial and final points of the path lie on the same horizontal line?
3. What is the force that does work when a body moves without friction over an inclined plane? Does this work depend on the length of the inclined plane?

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## 9.4.

### Potential Energy of a Body Acted Upon by the Force of Gravity

The formula

$$A = mg(h_1 - h_2), \quad (9.4.1)$$

expressing the work done by the force of gravity applied to a body can be represented in a different form. Opening the brackets and transposing the

terms, we get

$$A = -(mgh_2 - mgh_1). \quad (9.4.2)$$

The right-hand side of this formula contains the expression which is the *change in the quantity  $mgh$* <sup>11</sup>. The work done by the force of gravity is equal to this change, taken with the minus sign.

In Sec. 9.2, we called the quantity  $mv^2/2$ , whose change is equal to the work done by the force, the kinetic energy of a moving body. We now encounter another quantity whose change (but with the opposite sign) is also equal to the work done by the force (now the force of gravity). For this reason, the quantity  $mgh$  is also called energy, albeit *potential* and not kinetic. The quantity  $mgh$  is the potential energy of a body at a height  $h$  above the zero level.

Consequently, *the work done by the force of gravity is equal to the change in the potential energy, taken with the opposite sign*.

The minus sign in front of the change in the potential energy indicates that when the work done by the force of gravity is positive, the potential energy of the body decreases. On the contrary, when the force of gravity does a negative work i.e. when a body is thrown upwards, the potential energy of the body increases. The kinetic energy "behaves" in the opposite way.

Let us denote the potential energy  $mgh$  by  $E_p$ . Then we can write

$$A = -(E_{p_2} - E_{p_1}). \quad (9.4.3)$$

Let us assume that the height  $h_2$  in (9.4.2) corresponds to the zero level. We denote by  $h$  the height of the body above the zero level. Then  $E_{p_2} = mgh_2 = 0$ , and formula (9.4.3) becomes

$$E_p = A.$$

Hence it follows that *the potential energy of a body acted upon by the force of gravity is equal to the work done by this force when the body descends to the zero level*.

It should be recalled that similar definitions were given on p. 189 for kinetic energy, the zero level for which is the velocity  $v = 0$ .

Unlike kinetic energy, which depends on the velocity of motion of a body, potential energy does not depend on velocity. Hence, a body at rest can have a potential energy. *The potential energy depends on the position of the body relative to the zero level, i.e. on the coordinates of the body*, since the height  $h$  is just the coordinate of the body.

It was shown above that the zero level can be chosen arbitrarily. It may turn out that the body is below the zero level and has a negative coordinate. In this case, the potential energy of the body is also negative. *The sign and magnitude of the potential energy depends on the choice of the zero level*. On the other hand, the work done upon a displacement of a body is determined by the *change in its potential energy* and does not depend on the zero level.

<sup>11</sup> It should be recalled that the change in some quantity is the difference between the subsequent and the previous values and not vice versa.



Fig. 187

**THE POTENTIAL ENERGY OF AN ELEVATED BODY IS THE ENERGY OF INTERACTION.** While considering the potential energy of a body at a height  $h$  above the zero level, we "forgot" about the fact that the body possesses this energy due to its interaction with the Earth. In the absence of the Earth, there would be no force of gravity with which the Earth acts on the body and no potential energy  $E_p = mgh$ . Hence, the potential energy is said to be the *energy of interaction*. In this respect, it differs from the kinetic energy which can be called the *energy of motion*. Strictly speaking, the potential energy pertains to a system of bodies rather than to a single body. In the case under consideration, this system consists of the Earth and the body lifted above it. Therefore, the potential energy is the higher, the "stronger" the interaction between bodies, i.e. the stronger the force with which one body acts on the other.

**POTENTIAL ENERGY AND STABILITY OF EQUILIBRIUM.** There exists an interesting relation between the potential energy of a body (or a system of bodies) and the equilibrium of this body. This relation can be easily traced by using the example we have already considered in Sec. 7.3. In this section, it was shown that the ball on a concave support (see Fig. 164) is in a stable equilibrium. The equilibrium is stable since any small deviation from the position on the "bottom" gives rise to a force returning the ball to its original position.

Let us now find out what is the potential energy of the ball in different positions. In the lowest position (on the "bottom"), the potential energy of the ball is lower than in any of neighbouring positions (Fig. 187). This allows us to formulate the condition of equilibrium: *in a stable equilibrium, the potential energy is at the minimum*. The term "equilibrium" refers here to the body-Earth system.

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1. Give the definition of the potential energy of a body.
2. What is the relation between the work done by the force of gravity and the potential energy of a body?
3. How does the potential energy of a body moving upwards change?
4. What happens to the potential energy of a freely falling body?
5. What is the difference between the potential energy of an elevated body and its kinetic energy?
6. What can be said about the potential energy of a body in a stable equilibrium?

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### Exercise 30

1. A 2.5-kg load falls from a height of 10 m. What will be the change in its potential energy a second after the beginning of motion (the initial velocity of the load is zero)?
2. What is the work done by the force of gravity when a man having a mass of 75 kg climbs to the fifth floor, if the height of each storey is 3.0 m?
3. The difference in heights of the start and finish of the route at slalom competitions is 400 m. A slalomist takes off and successfully reaches the finish. What is the work done by the force of gravity acting on him if his weight at the start is 686 N?
4. The finishing post of a slalom route is 2000 m above the sea level, while the starting post is 400 m above the finish. What is the potential energy of the slalomist at the start relative to the finish and relative to the sea level? The mass of the slalomist is 70 kg.

### Hometask

Analyze Fig. 161 (p. 167) (a ball in an unstable equilibrium on the top of a convex support) and compare the potential energy of the ball at this and neighbouring positions. Formulate the condition of an unstable equilibrium.

## 9.5.

### Work Done by an Elastic Force. Potential Energy of a Body Subject to Elastic Deformation

It is well known that elastic force emerges upon deformation of bodies. Its magnitude is proportional to the deformation (elongation), while its direction is opposite to that of the displacements of points of the body under deformation.

Figure 188a shows a spring in its natural, undeformed state. The right end of the spring is fixed, while the left end is connected to a body. We direct the  $X$ -axis as shown in the figure. If we compress the spring by displacing its left end by the hand by  $x_1$  (Fig. 188b), this gives rise to an elastic force exerted by the spring on the body. The projection of this force onto the  $X$ -axis is

$$(F_{el1})_x = -kx_1,$$

where  $k$  is the rigidity of the spring.

Let us now leave the spring alone. Then its free end will shift to the left. During the displacement of the turns of the spring, the elastic force does a work. Let us calculate this work.

Let us suppose that the left end of the spring moves from position  $A$  to position  $B$  (Fig. 188c). In this position, the spring deformation is  $x_2$  and not  $x_1$ . The magnitude of the displacement of the end of the spring is equal to

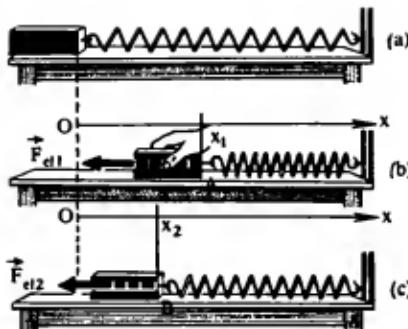


Fig. 188

the difference  $x_1 - x_2$  in the coordinates of its end. It can be seen that the directions of the force and displacement coincide. Therefore, to calculate the work done by the elastic force, we must multiply the magnitudes of the elastic force and displacement. But the elastic force varies as the body moves from point to point. At point A, the magnitude of the force is  $kx_1$ , while at point B it becomes  $kx_2$ .

To calculate the work done by the elastic force, we must multiply the average value of the magnitude of the force by  $x_1 - x_2$ :

$$A = F_{\text{el av}}(x_1 - x_2).$$

The elastic force is proportional to the deformation of the spring. Therefore, the average value of the magnitude of the elastic force can be defined as the arithmetic mean of its initial and final values:

$$F_{\text{el av}} = k \frac{x_1 + x_2}{2}.$$

It should be recalled that the average velocity in uniformly accelerated motion is defined in a similar way (see Home task on p. 54), the dependence of the instantaneous velocity on time is also linear.

In order to obtain the work done by the elastic force, we must multiply the value of this force by the magnitude  $x_1 - x_2$  of the displacement:

$$A = k \frac{x_1 + x_2}{2} (x_1 - x_2).$$

Since  $(x_1 + x_2)(x_1 - x_2) = x_1^2 - x_2^2$ , the formula for work becomes

$$A = \frac{k}{2} (x_1^2 - x_2^2).$$

This formula can also be written in the form

$$A = - \left( \frac{kx_2^2}{2} - \frac{kx_1^2}{2} \right). \quad (9.5.1)$$

The right-hand side of this expression contains the change in the quantity  $kx^2/2$  with the minus sign.

In Sec. 9.4, the quantity  $mgh$  whose change with the minus sign is equal to the work done by the force of gravity, was called the potential energy of an elevated body. Similarly, the quantity  $kx^2/2$  is called *the potential energy of a body subject to elastic deformation* (e.g., a spring).

Thus, formula (9.5.1) indicates that *the work done by an elastic force is equal to the change in the potential energy of the spring with the minus sign*.

If we denote, as before, the potential energy  $kx^2/2$  by  $E_p$ , we can write again

$$A = -(E_{p_2} - E_{p_1}). \quad (9.5.2)$$

Just like the value of  $mgh$ , the potential energy of a body subject to an elastic deformation depends on coordinates, since  $x_1$  and  $x_2$  in (9.5.1) are not only the elongations of the spring but also the coordinates of its end.

Formula (9.5.1) shows that the work done by the elastic force depends only on the initial and final coordinates. Therefore, we can repeat for the work done by the elastic force what has been said above about the work done by the force of gravity: it does not depend on the shape of the path, and if a body moves under the action of an elastic force along a *closed* path, the work done by this force is zero.

Let us assume that the coordinate of the end of the undeformed spring in formula (9.5.1) is zero ( $x_2 = 0$ ) and denote its elongation by  $x$ . Then  $E_{p_2} = kx^2/2 = 0$ , and (9.5.2) becomes

$$E_p = A.$$

Hence it follows that *the potential energy of a body subject to elastic deformation is equal to the work done by the elastic force when the body goes over to a state in which its deformation is zero*.

**THE POTENTIAL ENERGY OF A BODY SUBJECT TO ELASTIC DEFORMATION IS THE ENERGY OF INTERACTION.** It was mentioned in Sec. 9.4 that the potential energy  $mgh$  is the energy characterizing the interaction between the lifted body and the Earth. The potential energy of a body subject to elastic deformation, say, a spring, is also an interaction energy. But now we mean the interaction between individual particles constituting the body. If the elastic body is a spring, there is an interaction between its turns (the particles of the material of which the spring is made). The expression  $mgh$  for potential energy includes the force of gravity  $mg$ . The expression  $kx^2/2$  for the potential energy of a body subject to elastic deformation includes the elastic force  $kx$ . Therefore, potential energy is generally an interaction energy.

---

1. In what way is the average elastic force defined?
2. In which respect are the works done by elastic force and by the force of gravity similar?
3. What is the work done by an elastic force if a body on which it acts returns to the original position after covering a certain distance?

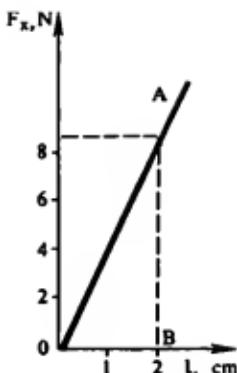


Fig. 189

4. What is the value of the potential energy of a body subject to elastic deformation?
5. What do the potential energies of a body acted upon by the force of gravity and a body subject to elastic deformation have in common?

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### Exercise 31

1. The maximum force with which a boy can stretch a dynamometer is 400 N. What is the work done by this force in stretching, if the rigidity of the dynamometer spring is 10 000 N/m?
2. An 18-kg body is suspended from a spring with a fixed upper end. The length of the stretched spring is 10 cm. When a 30-kg body is suspended from it, its length becomes 12 cm. Calculate the work done by the external force in stretching the spring from 10 to 15 cm. What is the work done by the elastic force in this case?
3. Figure 189 shows the dependence of the elastic force emerging upon compression of the spring of a toy-pistol on its deformation. Calculate the work done by an external force upon a compression of the spring by 2 cm. Prove that this work is equal in magnitude to the area of the triangle  $AOB$ .
4. Two springs have the same rigidity. One of them is compressed by 5 cm, while the other is stretched by 5 cm. What are the differences in the elongations of the springs and in their potential energies?
5. A load is attached to a spring balance. When released, the load went down so that the pointer stopped against figure 3. What is the increase in the potential energy of the spring of the balance if the spring balance is graduated in newtons and the distance between two adjacent divisions on the scale is 5 mm.

6. A compressed spring whose rigidity  $k = 10\,000 \text{ N/m}$  acts on a body attached to it with a force of 400 N. What is the potential energy of the spring? What is the work done by an external force for its compression? What is the work done by the elastic force of the spring when it is released?

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## 9.6.

### The Law of Conservation of Total Mechanical Energy

At the beginning of the chapter it was mentioned that the energy obeys the law of conservation. Let us elucidate the meaning of this law.

Let us consider how the energy of bodies interacting *only with one another* changes. It should be recalled that such bodies form a closed system (see Chapter 8).

Interacting bodies may have kinetic and potential energy simultaneously. For example, an artificial satellite of the Earth has a kinetic energy as a moving body. Besides, the satellite-Earth system has a potential energy since the satellite and the Earth interact through the force of universal gravitation. Colliding balls simultaneously have kinetic energy due to their motion and potential energy since they are subject to elastic deformation.

If bodies forming a closed system interact they move relative to one another. In this motion, both their velocities and coordinates may change. Consequently, the kinetic energy of the bodies, as well as their potential energy, may change.

Let us denote by  $E_p$ , the potential energy of interacting bodies at a certain instant of time and by  $E_k$ , their total kinetic energy at the same instant. The potential and kinetic energies of the same bodies at some other instant of time will be respectively  $E_{p_2}$  and  $E_{k_2}$ .

In Secs. 9.4 and 9.5 it was established that when bodies interact through an elastic force or a force of gravity, the work  $A$  done by these forces is equal to the change in the potential energy of the bodies with the opposite sign:

$$A = -(E_{p_2} - E_{p_1}). \quad (9.6.1)$$

On the other hand, according to the kinetic energy theorem, the work done by the same force is equal to the change in the kinetic energy:

$$A = E_{k_2} - E_{k_1}. \quad (9.6.2)$$

Comparing (9.6.1) and (9.6.2), we see that the change in the kinetic energy is equal in magnitude to the change in the potential energy, but they have opposite signs:

$$E_{k_2} - E_{k_1} = -(E_{p_2} - E_{p_1}). \quad (9.6.3)$$

If the potential energy of bodies increases, their kinetic energy decreases by

the same value, and vice versa. Hence it is clear that one type of energy is converted into the other.

Obviously, formula (9.6.3) can be written in a different form:

$$E_{k_2} + E_{p_2} = E_{k_1} + E_{p_1}. \quad (9.6.4)$$

Thus, the sum of the kinetic and potential energies of bodies constituting a closed system and interacting through forces of universal gravitation or elastic forces remains constant. This is the essence of the *law of conservation of energy*.

The sum of the kinetic and potential energies of a system of bodies is usually called the *total mechanical energy*.

The total mechanical energy of a closed system of bodies interacting through the forces of gravity or elastic forces remains unchanged.

The conversion of potential energy into kinetic energy and vice versa is one of the most remarkable phenomena in nature. This is the main feature of energy.

The law of conservation and transformation of energy allows us to understand better the physical meaning of work. The fact that the same work leads to an increase in kinetic energy and to the same decrease in potential energy implies that *the work is the energy transformed from one form to another*.

In Chapter 8, we considered the law of conservation of momentum in a closed system. Now, we obtained the second conservation law, viz. the law of conservation of energy. These two laws are of the most general nature and perfectly true even when the laws of Newtonian mechanics become invalid.

The law of conservation of energy can be used for solving many problems in mechanics. It allows one to solve problems easier than with the help of Newton's laws.

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1. What is called the total mechanical energy of a body?
2. What is the essence of the law of conservation of the total mechanical energy of a body moving under the action of the force of gravity?
3. What is the essence of the law of conservation of the total mechanical energy of a body moving under the action of an elastic force?
4. Is the law of conservation of the total energy of a body (or a system of bodies) observed if the force of gravity and the elastic force act simultaneously?
5. A satellite moves in a circular orbit around the Earth. Using a rocket engine, it was transferred to another orbit. Has its total energy changed?

## EXAMPLES OF SOLVING PROBLEMS

1. Which height  $h$  will be attained by a body thrown upwards at an initial velocity  $v_0$ ?

**Solution.** Let us take the zero level at the point from which the body was thrown. At this point, the potential energy of the body is zero, while its kinetic energy is  $mv_0^2/2$ . Hence the total energy of the body is  $0 + mv_0^2/2 = mv_0^2/2$ . At the uppermost point, i.e. at the height  $h$ , the potential energy of the body is  $mgh$ , while its kinetic energy is zero. Consequently, the total energy of the body at the uppermost point is  $mgh$ . According to the law of conservation of the total energy, we have

$$mgh = \frac{mv_0^2}{2},$$

whence

$$h = \frac{v_0^2}{2g}.$$

The same result has been obtained by us earlier in a more intricate way (see p. 124).

2. A ball whose mass  $m = 3$  kg was at a height  $h = 3$  m above a platform fixed on a spring (Fig. 190a). Determine the maximum compression  $l$  of the spring caused by the falling of the ball on the platform (Fig. 190b), if the rigidity  $k$  of the spring is  $700$  N/m. The masses of the spring and platform can be neglected.

**Solution.** We shall assume that the potential energy of the ball lying on the platform at the maximum compression of the spring (zero level) is zero. Then the potential energy of the ball at the initial instant is

$$E_{p_1} = mg(h + l).$$

The kinetic energy of the ball and the spring at this instant is zero. Consequently, the total energy  $E_1$  of the ball-spring system at the initial instant is determined by the potential energy of the ball:

$$E_1 = E_{p_1} = mg(h + l).$$

When the compression of the spring is maximum, the kinetic energy of the ball is zero, and the spring has the potential energy of  $kl^2/2$ . Hence, the total energy  $E_2$  of the same system at the moment when compression is maximum is

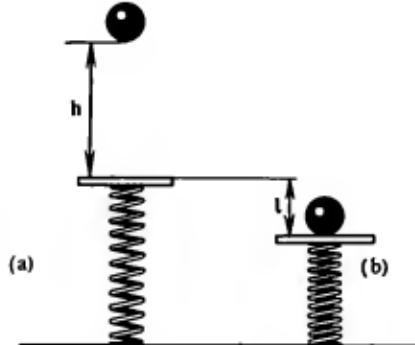


Fig. 190

$$E_2 = \frac{kl^2}{2}.$$

According to the energy conservation law, we have

$$E_1 = E_2,$$

or

$$mg(h + l) = \frac{kl^2}{2}.$$

Solving the obtained quadratic equation and substituting the data of the problem, we obtain  $l \approx 0.5$  m.

### Exercise 32

1. A body falls from a certain height above the ground. At the moment it strikes the ground, its velocity is 30 m/s. From what height does the body fall?
2. A shell that has acquired at the shot from the gun the initial velocity of 280 m/s flies vertically upwards. At what height above the place of the shot will its kinetic energy be equal to the potential energy?
3. A 2.0-kg body falls from the height of 30 m above the ground. Calculate the kinetic energy of the body at the moment when it is 15 m above the ground and at the moment it hits the ground.
4. The beetle-head of a pile-driver, falling from the height of 8 m, has a kinetic energy of 18 000 J at the moment of impact. What is its mass?
5. A stretched spring, while contracting, entrains a 50-g body over a horizontal plane without friction. When the spring deformation becomes zero, the velocity of the body is 5 m/s. What is the elongation of the spring, if its rigidity is 10 000 N/m?
6. A 400-g body is attached to a compressed spring whose rigidity is 100 N/m. When the spring is released, the body vibrates so that the maximum elongation of the spring is 10 cm. What is the maximum velocity of the vibrating body? (The spring mass should be neglected.)
7. A 50-g ball is moving at a velocity of 10.0 m/s and collides with a ball at rest whose mass is 110 g. What are the velocities of the balls after the collision? Assume that they move along the straight line connecting their centres.

*Hint.* Use the laws of conservation of energy and momentum for solving this problem. The sums of the kinetic energies and of the projections of momenta onto the axis passing through the centres of the balls must be the same before and after the collision.

## 9.7.

### Friction Work and Mechanical Energy

We are left now with the third mechanical force whose work has to be considered, viz. sliding friction. Friction is manifested to a certain extent in any motion occurring on the Earth. What is the difference between the friction work and the work done by other forces in mechanics?

Friction appears only during relative motion of bodies in contact. If one of them is assumed to be fixed, the direction of the force acting on the other body is always opposite to the direction of its velocity. Friction does not depend on coordinates, i.e. on the mutual arrangement of the bodies.

Therefore, the work of friction cannot be represented as a change in a certain potential energy. However, it can be calculated on the basis of the theorem on kinetic energy:

$$A = \frac{mv_2^2}{2} - \frac{mv_1^2}{2}.$$

Since friction is directed against the velocity vector,  $v_2 < v_1$  and the work  $A$  is negative.

When the force of gravity or an elastic force acts on a body, it can move *opposite* to the direction of the force (for example, a body thrown upwards) as well as *in* the direction of the force (a freely falling body). In the former case, the work done by the force is negative, while in the latter case it is positive. If a body moves "back and forth", the total work is zero.

The situation for the work of friction is different. Friction is directed against the relative velocity of interacting bodies. For this reason, *the work done by friction during the motion of a body over a closed path differs from zero*.

A body thrown upwards starts to move against the force of gravity which in this case does a negative work. Therefore, its kinetic energy decreases. Having reached the uppermost point of the trajectory, the body stops for a moment and starts moving downwards.

If we push a body lying on a horizontal surface, it starts to move against the emerging friction which, like the force of gravity in the above example, does a negative work. The kinetic energy of the body decreases. After covering a certain distance, the body stops in this case too, but not "for a moment" as in the example with the body thrown upwards. It will stop forever and will not move back.

As a matter of fact, in the former example the kinetic energy gradually decreases, being converted into potential energy which is then again transformed into kinetic energy. On the other hand, when a body moves over a horizontal surface under the action of friction, its kinetic energy decreases but is not converted into potential energy. For this reason, the body does not move in the opposite direction after coming to a halt, since there is no energy at the expense of which the work could be done in such

a motion. The mechanical energy of the moving body was not transformed into some other kind of mechanical energy, it just "vanished".

**MECHANICAL ENERGY IS NOT ALWAYS CONSERVED.** It turns out that when friction (alone or together with other forces) acts on a body the law of conservation of mechanical energy is violated: the kinetic energy decreases while no potential energy appears instead. Thus, the total mechanical energy decreases.

Such a decrease in the total mechanical energy is observed even when a body falls to the ground in air and not in vacuum. During this motion, the potential energy of the body decreases by  $mgh$ , just as in vacuum. However, the velocity of the body at the moment it reaches the Earth surface is lower than for a free fall in vacuum. Its kinetic energy will also be lower. It will not be equal to the decrease in the potential energy. The work against the resistance of air is done at the expense of the lost energy. Although we know where the mechanical energy has been lost, it has still vanished, and the energy conservation law seems to have been violated.

However, the violation of the law of conservation of energy is only apparent here. As a matter of fact, friction of a body against another body leads to heating of the two bodies i.e. to an increase in their temperatures. It was mentioned in *Junior Physics* that the temperature of bodies is determined by the velocity of molecules constituting them, and hence by their kinetic energy. Therefore, the energy of motion of molecules of bodies being heated due to friction or the *internal energy* of the bodies increases. Is this increase in the internal energy just due to the lost kinetic energy of the entire body? Thorough measurements revealed that when the kinetic energy of moving bodies decreases due to friction, their internal energy (the energy of motion of molecules in a body) actually increases by just the same value. Consequently, although the mechanical energy decreases, it is not lost completely but only transforms to the energy of moving molecules.

Thus, we arrive at a very important conclusion that not only the mutual conversion of potential and kinetic energies is possible. *Mechanical energy can be converted into a nonmechanical form of energy, viz. the internal energy of moving particles constituting the body.* The remarkable feature of energy is that it may have different forms (kinetic, potential, internal and many others) that will be considered later. The law of conservation of energy just states that in a closed system, the sum of all kinds of energy is conserved. Each time when a loss of energy is observed in any process or phenomenon, we can be sure that some other kind of energy has appeared in the process.

**FRICITION IN ENGINEERING.** It should be recalled that the work done by forces applied to a body is always equal to the change in its kinetic energy (see Sec. 9.3). When friction is present, a fraction of the mechanical energy of the body is transformed into nonmechanical, internal energy, which leads to the heating of rubbing bodies. This energy is lost when useful work is considered. For this reason, friction always has to be reduced in any device (machine) designed for doing mechanical work. The designers and users of these machines should bear this circumstance in mind. If a machine, an engine or an apparatus are heated too much during operation, this is a sure sign that the friction is too strong and that measures should be taken

to decrease it. It was mentioned in Sec. 5.7 that lubrication is used for this purpose. We speak about excessive heating since friction in mechanical equipment cannot be completely eliminated. A certain heating, and hence some loss of energy is unavoidable. However, it must be made as small as possible.

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- 9
- 1. Friction acts on a body. Can the work done by this force be zero?
- 2. If a body acted upon by friction returns to the initial point after covering a certain distance, is the work of friction equal to zero?
- 3. How does the mechanical energy of a body which is acted upon sliding friction only change?

---

### Exercise 33

- 1. A 60-kg sledge, after sliding down the hill, covered a distance of 20 m over the horizontal part of the road. Find the work done by friction over this distance, if the coefficient of friction between the sledge runners and the snow is 0.020.
- 2. A part being sharpened is pressed with the force of 20 N to a whetstone whose radius is 20 cm. Calculate the work done by the motor during 2 min, if the whetstone speed of rotation is 180 rpm, and the coefficient of friction between the part and the whetstone is 0.30.
- 3. The driver of a motorcar puts the engine out of gear and applies the brakes at 20 m to a traffic light (the road is horizontal). Find the maximum velocity at which the car has enough time to come to a halt, if the mass of the car is 1.6 t and the force of friction is 4000 N.
- 4. A body moving over a horizontal plane experiences a friction of 100 N over a distance of 15 m. What is the change in its mechanical energy? Which energy (kinetic or potential) has changed?
- 5. A parachutist whose mass is 70 kg moves after a jump from a plane first with an acceleration and then, starting from the height  $h = 1000$  m and till landing, uniformly. What is the work done by the resistance of air during the uniform motion?
- 6. A 2-kg body falls from a height of 240 m and penetrates the ground to a depth of 0.2 m. The friction of the body against the ground is 10000 N. Was the motion of the body in air a free fall?
- 7. A 10-g bullet flying horizontally at a velocity of 600 m/s strikes a wooden block whose mass is 2.0 kg and sticks in it. As a result, both the bullet and the block are heated. Which part of energy is being converted into heat? The air drag should be neglected.

*Hint.* While solving this problem, make use of the laws of conservation of momentum and energy.

### Hometask

Give one of many examples of a situation when the total mechanical energy of a body or a system of bodies is not conserved.

## 9.8.

### Power

It should be recalled (see *Junior Physics*) that an engine producing work is characterized by a special quantity called *power*.

*The power of an engine or a mechanism is the ratio of the work done to the time interval over which this occurred.*

If we denote power by  $N$ , it is given by

$$\underline{N = \frac{A}{t}}. \quad (9.8.1)$$

This formula shows that the unit of power in SI is 1 J/s (joule per second). This unit is called a watt (W):

$$1 \text{ W} = 1 \text{ J/s}.$$

This is a comparatively small unit. In engineering, a 1000 times larger unit is often used. This is a kilowatt (kW). Sometimes, a unit that is million times larger than the watt is used, called a megawatt (MW).

Let us consider an example. At Krasnoyarsk hydroelectric station, a 5000-m<sup>3</sup> water flow (whose mass is  $5 \times 10^6$  kg) falls from the 100-m dam every second. Obviously, the power of the station is equal to the work done by the force of gravity on this mass of water per second:

$$N = \frac{mgh}{t} = \frac{m}{t} gh.$$

Taking into account that  $m/t = 5 \times 10^6$  kg/s, we get

$$N = 5 \times 10^6 \frac{\text{kg}}{\text{s}} \times 9.8 \frac{\text{m}}{\text{s}^2} \times 100 \text{ m} \simeq 5 \times 10^9 \text{ J/s}$$
$$= 5 \times 10^6 \text{ kW}.$$

If the power  $N$  is known, the work  $A$  done in time  $t$  is given by

$$A = Nt.$$

Hence it follows that we can take the work done per second at a power of 1 W as the unit of work. This unit of work is called a watt-second (W·s):

$$1 \text{ W} \cdot \text{s} = 1 \text{ J}.$$

However, a joule, and hence a watt-second, are too small units. Larger units are commonly used, e.g. a kilowatt-hour (kW·hr) or a megawatt-hour (MW·hr):

$$1 \text{ kW} \cdot \text{hr} = 1000 \text{ W} \times 3600 \text{ s} = 3.6 \times 10^6 \text{ W} \cdot \text{s} = 3.6 \times 10^6 \text{ J.}$$

$$1 \text{ MW} \cdot \text{hr} = 1000000 \text{ W} \times 3600 \text{ s} = 3.6 \times 10^9 \text{ W} \cdot \text{s}$$
$$= 3.6 \times 10^9 \text{ J.}$$

Aeroplanes, ships, rockets, motorcars and other transport facilities often move at a constant velocity. This means that forces developed by their engines are equal and opposite to drag forces. What determines the velocity of motion of these bodies?

We shall show that the velocity is determined by the engine power. Indeed,  $N = A/t$ . But  $A = Fs$ , where  $F$  is the magnitude of the resistance force.

Consequently,

$$N = \frac{Fs}{t}.$$

The ratio  $s/t$  is equal to  $v$ , where  $v$  is the magnitude of the velocity of motion of the body. Therefore,

$$N = Fv \quad (9.8.2)$$

or

$$v = \frac{N}{F}.$$

This formula shows that at a constant resistance, the velocity of a body is proportional to the engine power. For this reason, high-speed trains and motorcars require high-power engines. In many cases, however, the drag force is actually not constant but increases with velocity.

It was shown in Sec. 5.7 that when ships or aeroplanes move at high velocities, the resistance of air and water (fluid friction) is proportional to the square of velocity. This can be expressed by the formula  $F = \beta v^2$ , where  $\beta$  is the proportionality factor.

Substituting into formula (9.8.2) the quantity  $\beta v^2$  for  $F$ , we obtain the following expression for power:

$$N = \beta v^3.$$

Consequently, the power of aeroplane and ship engines is proportional to the third power of velocity. If, for example, the velocity of a plane has to be doubled, the power of its engines should be increased by a factor of eight. For this reason, each time when designers succeed in increasing the velocity of a new plane, this costs them a lot of efforts.

The formula  $F = N/v$  also shows that when the power  $N$  of an engine is constant, the force applied to a moving body due to the engine operation is stronger at low velocities than at high velocities. This is why the driver of a car shifts a lower gear while moving up the hill, where the maximum tractive force is required.

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- 1. What is power?
- 2. Is power a scalar or a vector quantity?
- 3. What determines the velocity of a uniformly moving body driven by an engine?
- 4. Which units of power are used in engineering and in every-day life? What is the relation between them?
- 5. Which physical quantity is expressed in kilowatt-hours?

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## EXAMPLES OF SOLVING PROBLEMS

- 1. What average power can be developed by a man having a mass of 70 kg when he runs upstairs to the height of 10 m in 15 s?

*Solution.* When the man goes upstairs, he does the following work against force of gravity:

$$A = mgh.$$

Consequently, the power developed by the man is

$$N = \frac{mgh}{t}.$$

Substituting into this formula the values of the quantities from the condition of the problem, we get

$$N = \frac{70 \text{ kg} \times 9.8 \frac{\text{m}}{\text{s}^2} \times 10 \text{ m}}{15 \text{ s}} \simeq 460 \text{ W.}$$

- 2. A crane with a 12-kW engine lifts a load at a constant velocity of 90 m/min. What is the mass of the load?

*Solution.* Using the formula

$$N = Fv$$

for power, we can express the force exerted by the crane on the load being lifted:

$$F = \frac{N}{v}.$$

When the motion of the load is uniform, the magnitude of this force is  $mg$ . Therefore,

$$mg = \frac{N}{v}$$

or

$$m = \frac{N}{vg}.$$

Substituting into this formula the values of the quantities from the condition of the problem, we obtain

$$m = \frac{12000 \text{ W}}{9.8 \frac{\text{m}}{\text{s}^2} \times 1.5 \frac{\text{m}}{\text{s}}} \simeq 820 \text{ kg.}$$

---

### Exercise 34

1. An aeroplane uniformly flies along the straight line at a velocity of 900 km/hr. What is the drag force if the engines of the plane develop a power of 1800 kW?
2. A crane with an 8-kW engine lifts a load at a constant velocity of 6 m/min. What is the mass of the load?
3. A shaft is processed on a lathe. The power developed by the engine is 3 kW. What amount of work is done if the shaft is processed for 2 min?
4. What is the work done by a hydroelectric station during one year if the power of its generators is 2.5 MW?
5. A 2000-kg motorcar moves along a horizontal road at a velocity of 72 km/hr. The resistance to its motion is equal to 0.05 of its weight. Calculate the power developed by the engine.

---

## 9.9.

### Energy Transformation. Utilization of Machinery

About two hundred years have passed since various machines began to be widely used by people. These machines are driven by engines which, in turn, receive energy from some sources. From the point of view of mechanics, machines are used as an aid to do work by some forces. However, to do work means to spend energy equal at least to this work. Nowadays, the main types of energy due to which work is done are the energy liberated upon combustion of fuel (coal, oil or gas), the energy of falling water and the so-called nuclear energy released in nuclear reactors. None of these types of energy is directly supplied to machines.

On its way to the machines where work is done, energy undergoes transformation from one type to another. For example, the energy of interaction among particles of fuel and oxygen (potential energy) is first transformed into the internal energy of the particles formed during the burning of fuel. Then, this energy is supplied by heat transfer to steam in a steam turbine which drives an electric generator where the mechanical energy of rotation is transformed into electric energy. This is the principle of operation of a thermal power plant. The energy is transmitted by cables from the generator to electric motors installed on various lathes and other mechanisms. In motors, electric energy is again converted into mechanical energy and is supplied to lathes and other machines with the help of various transmissions like levers, inclined planes, screws and pulleys.

We have considered a chain of transformations of energy "on its way" from the furnace of a thermal power plant to a machine. It should be also noted that fuel itself has appeared on the Earth as a result of a complex chain of energy transformations. The origin of this chain is the Sun which is the source of life on the globe.

It should be emphasized that *these transformations* (we have listed just a few of them) *obey the law of energy conservation* which implies that upon any transformation, it is impossible to receive more energy of one type than the consumed amount of energy of another type. No engine can give more mechanical energy than the electric or internal energy supplied to it. There is no engine which can do more work than the amount of energy spent.

On the contrary, a part of energy is inevitably lost in real engines due to friction. It is lost in the sense that it is transformed due to friction into internal energy, thus causing the engine heating. Similarly, the work done by forces operating in a machine is always somewhat smaller than the energy spent.

**ON PERPETUAL MOTION MACHINES.** All what has been said above became known only after the law of energy conservation had been discovered in the middle of the 19th century. Before that, efforts were made for centuries to design a machine which would give more work than the energy spent. It was even called beforehand a perpetual motion machine (*perpetuum mobile*). However, such a machine could not be created. Moreover, it cannot be made even in principle.

Figure 191 shows a diagram of one of the numerous projects of a perpetual motion machine. It consists of two wheels (pulleys) located in the upper and lower parts of a tower filled with water. The pulleys are connected by an endless rope with light empty boxes fixed to it at certain intervals. It can be seen that at each instant some of the boxes are submerged into water

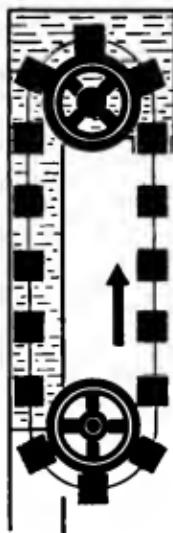


Fig. 191

while the others are in air. The author of the project was sure that the right-hand (in the figure) boxes would make the pulley rotate as they float up to the surface under the action of buoyancy. The boxes that float up are replaced by others, and "perpetual" motion is sustained. Rotating wheels could drive electric generators, thus producing energy "free of charge" and in unlimited amounts, since the device operates "perpetually".

Actually, however, the design has some faults, and such an engine cannot work. Indeed, while some boxes are floating up, the others, on the contrary, sink. The boxes must move against the buoyancy force. Besides, as they enter water from below, they are acted upon by the entire water column. The force of pressure of this column is stronger than the buoyancy force.

Similar errors can be found in any design of a perpetual motion machine. Attempts to create such a device are doomed to failure because the energy conservation law "prohibits" the performance of more work than the energy spent.

It is interesting to note that even at present there are "inventors" who do not give up fruitless attempts to create a perpetual motion machine.

The goal of engineering consists not in circumventing the energy conservation law but in decreasing energy losses in machines, engines and generators.

?

1. What are generators, engines and machines intended for?
2. What is the essence of the idea behind a perpetual motion machine? Why is this idea impracticable?
3. Which energy transformations take place when a rifle is fired or when a rocket is launched?

## 9.10

### Efficiency

When work is done in a machine due to expenditure of energy, one should distinguish between the *useful work* and the *total work done*.

Useful work is the work for which a machine is designed and used. For example, the useful work of a crane is the work done to lift a load. For a lathe, it is the work done to overcome the friction between the job and the cutting tool, and so on.

However, the useful work in any engine is always less than the total work since there always exists friction whose negative work leads to the heating of different parts of the machine or engine. Heating cannot be considered as a useful result of the machine operation, since it cannot be used for doing mechanical work. As a result of heating, a fraction of energy supplied to the engine is transformed not into mechanical energy but into internal energy and hence is not used for doing work.

Therefore, every machine, engine or mechanism is characterized by a special quantity, determining the efficiency of utilization of the energy supplied to it. It should be recalled that (see *Junior Physics*, Sec. 66) this quantity is called the *efficiency*.

We can also speak about the efficiency of a generator in which one type of energy is transformed into another. For example, mechanical energy is transformed in an electric generator into work by an electric current. Due to friction and other reasons, the work of electric current is always somewhat smaller than the mechanical energy spent by the turbine.

*The generator efficiency is the ratio of the useful work to the energy spent*

The efficiency of a machine can never be greater than unity. In real machines, engines and generators, it is always less than unity due to inevitable energy losses caused above all by the negative work of friction. However, there are some other, non-mechanical causes of energy losses.

It should be emphasized once again that the word "loss" does not mean that energy vanishes completely. It only means that a fraction of energy is transformed into some type of energy which is not the required one and is no longer available for utilization.

Efficiency is expressed as a percentage. If we denote efficiency by  $\eta$  (the Greek letter "eta"), the useful work (or energy) by  $A_u$  and the total work done (energy spent) by  $A_s$ , we have

$$\eta = \frac{A_u}{A_s} \cdot 100\%.$$

?

1. What is the chain of energy transformations that leads to an increase in the internal energy of the heating element in an electric stove if electric power is supplied to the circuit from a hydroelectric station? Start with the Sun.
2. A body has fallen to the ground from a certain height. Where has its potential energy gone?
3. A blacksmith raised a hammer and struck an ingot on the anvil. Which energy transformations occur in this case?
4. A deformed metallic spring is immersed into an acid dissolving the metal of which the spring is made. What happens to the potential energy of the spring after its dissolution?

## EXAMPLES OF SOLVING PROBLEMS

1. A crane is driven by an engine whose power is 10 kW. What time does it take to lift a 2-t load to a height of 50 m, if the efficiency of the engine is 75%?

*Solution.* The crane does the useful work given by

$$A_u = mgh.$$

The total work done  $A_s$  is expressed by the formula

$$A_s = Nt,$$

where  $N$  is the engine power and  $t$  is the time of operation of the crane.

According to the condition of the problem, the useful work amounts to

only 75% of the work done by the engine. Therefore,

$$A_u = 0.75 \text{ Nt.}$$

Hence

$$t = \frac{A_u}{0.75 \text{ N}} = \frac{mgh}{0.75 \text{ N}},$$

$$t = \frac{2000 \text{ kg} \times 9.8 \frac{\text{m}}{\text{s}^2} \times 50 \text{ m}}{0.75 \times 10^4 \frac{\text{J}}{\text{s}}} \approx 130 \text{ s.}$$

2. A 2-t motor car with the brakes applied descends at a constant velocity down a mountain road and reaches a point whose altitude is 80 m lower than that of the starting point. What amount of energy  $Q$  has been liberated in the brakes?

*Solution.* If the brakes were not applied, the decrease in the potential energy would be equal to the increase in the kinetic energy. But since the car was moving at a constant velocity, the kinetic energy did not increase during the descent. Consequently, the entire potential energy lost was transformed into the internal energy, i.e.

$$Q = mg(h_1 - h_2).$$

Substituting the numerical data, we get

$$Q = 2000 \text{ kg} \times 9.8 \frac{\text{m}}{\text{s}^2} \times 80 \text{ m} \approx 1.6 \times 10^6 \text{ J.}$$

---

### Exercise 35

1. A crane is driven by an engine whose power is 7.36 kW. Find the mass of the load lifted by the crane at a velocity of 6 m/min, if the engine efficiency is 80%.
2. An aeroplane is flying in a straight line at a uniform velocity of 800 km/hr. Find the tracing force of its motors if their power is 1800 kW. Assume that the efficiency is 70%.
3. A pump with a 3 kW motor lifts water from a 20 m-deep well. Find the mass of the water lifted in two hours if the efficiency of the pump is 70%.
4. Every second, 170 t of water fall from the 30-m dam of a hydroelectric station. The electric power generated by the station is 10 MW. What is the efficiency of converting the energy of falling water into electric energy?

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## 9.11. Flow of Fluid in Pipes. Bernoulli's Law

In this section, we shall apply the energy conservation law to the flow of fluid in pipes. This type of motion is often encountered in everyday life. Water is fed to consumers through the pipes of a water-supply system. Pipes are used for supplying lubricating oil to machines and fuel to engines. Flow of fluid in pipes also occurs in nature. It is sufficient to say that the blood circulation of animals and human beings is the flow of blood in pipes, viz. blood vessels. To a certain extent, the flow of water in rivers is also a modification of the fluid flow in pipes. A river bed serves as a pipe for flowing water.

It is well known that in accordance with Pascal's law, a fluid *at rest* in a vessel transmits an external pressure to all points of the fluid without any change. If, however, a fluid flows *without friction* through a pipe with a variable cross-sectional area, experiments show that the pressure turns out to be different along the pipe. Let us now find out why the pressure in a fluid flow depends on the cross-sectional area of the pipe. But first let us consider an important feature of any fluid flow.

**VELOCITY OF A FLUID AND CROSS-SECTIONAL AREA OF THE PIPE.** Let us suppose that a fluid flows in a horizontal pipe with a varying cross-sectional area, for example, in the pipe a part of which is shown in Fig. 192.

If we mentally draw several cross sections of areas  $S_1$ ,  $S_2$  and  $S_3$  along the pipe, and measure the volume of fluid flowing through each section in a certain time  $t$ , we find that the same volume of fluid passes through every cross section. This means that all the fluid that passes in time  $t$  through the first cross section passes through the third cross section over the same time, although the area of this cross section is considerably smaller than that of the first. If this were not the case and, for example, a smaller amount of fluid passed through the cross-sectional area  $S_3$  than through  $S_1$  over the time  $t$ , an excess of fluid would be accumulated somewhere. But the fluid fills the entire pipe and has no place to be accumulated. Here we assume that a given mass of the fluid always has the same volume; it cannot be compressed and hence decrease in volume (the fluid is said to be *incompressible*).

But how can a fluid that has passed through a wide section "squeeze" through a narrow cross section in the same time? Obviously, in order to be able to do that, *the velocity of the fluid must be higher in the narrow parts of the pipe than in the wide parts*. It is well known, for example, that in the narrow parts of a river bed the fluid velocity is higher than in the wide parts.

**VELOCITY AND PRESSURE.** Since the velocity of a fluid flow through a pipe increases as it passes from a larger cross-sectional area to a smaller one, fluid moves with *an acceleration*. In accordance with Newton's second



Fig. 192

Daniel Bernoulli (1700-1782) was a specialist in mathematics and mechanics. From 1725 to 1733, he worked in the Russian Academy of Sciences where he was engaged in research in the fields of mathematics, mechanics, as well as physiology. It was here that he wrote the book "Hydrodynamics" which contains the derivation of the equation describing the flow of an ideal fluid and known as Bernoulli's law.



law, this means that a *force acts on the fluid*. What kind of force is it?

This can be only the difference between the forces of pressure in the wide and narrow sections of the pipe. Thus, the pressure in the wide section of a pipe should be higher than in the narrow one.

This conclusion can also be drawn from the energy conservation law.

Indeed, if the velocity of a flow increases in narrow sections, its kinetic energy also increases. Since we assume that the fluid flows without friction, this increase in the kinetic energy must be balanced by a decrease in the potential energy because the total energy must remain constant. What sort of potential energy is meant here? If the pipe is horizontal, the potential energy of interaction with the Earth is the same in all sections of the pipe and it cannot change. Consequently, only the energy of elastic interaction is left. The force of pressure that makes the fluid flow through the pipe is just the elastic force of the compressed fluid. When a fluid is said to be incompressible, it only means that it cannot be compressed so that its volume noticeably changes, but a very small compression causing the elastic forces inevitably occurs. These forces are responsible for creating a pressure in a fluid. It is this compression of the fluid that decreases in narrow parts of the pipe. Therefore, the pressure in narrow cross sections of pipes should be lower than in wide sections.

This is the essence of the law discovered by the Petersburg Academician DANIEL BERNOULLI.

The pressure of a fluid flow is higher in sections where the fluid velocity is lower. On the contrary, the pressure is lower in sections where the velocity is higher.

The above reasoning refers to the case when a fluid in a pipe can be considered as a closed system. The energy conservation law is valid only for such a system. In reality, a fluid flows in a pipe due to a pressure drop under



Nikolai I. Gorovich Zhukovskii (1847-1921) was a Russian scientist specializing in the field of mechanics. He was the founder of modern aerohydrodynamics whose growth is inseparably linked with the progress in the construction of aircraft. In 1918, he founded the Central Aerohydrodynamic Institute (TsAGI) and became its first Director. Zhukovskii carried out numerous fundamental investigations in the fields of mechanics of rigid bodies, astronomy, mathematics, hydrodynamics, etc. He also wrote several textbooks on theoretical mechanics.

the action of an external force. In this case, the energy changes and, as a rule, its change is equal to the work done by the external force. However, the Bernoulli law remains valid in this case as well.

If a pipe through which a fluid is flowing is fitted with open tubes—*manometers* (Fig. 193), it is possible to observe the pressure distribution along the pipe. In narrow sections, the height of the liquid column in a manometer is lower than in wide sections. This means that the pressure is lower here.

The smaller the pipe cross section, the higher the flow velocity in it and the lower the pressure. Obviously, it is possible to select a cross section where the pressure is equal to the atmospheric pressure (in this case, the height of the liquid column in the manometer is zero). If we take a still

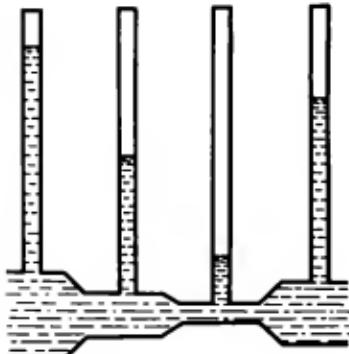


Fig. 193

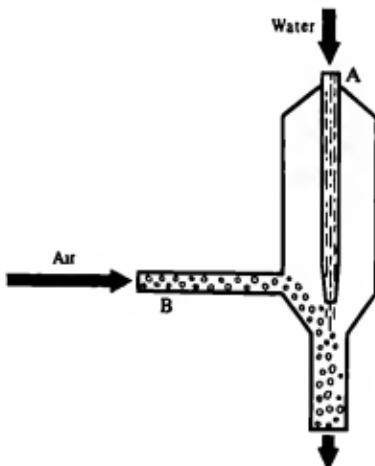


Fig. 194

smaller cross section, the pressure of the liquid in it will be lower than the atmospheric pressure.

Such a flow of liquid can be used for pumping air out of a vessel. A jet pump operates on this principle (its schematic diagram is shown in Fig. 194). A jet of water flows through a pipe *A* with a spout. The pressure of water at the spout is lower than the atmospheric pressure. For this reason, the air being pumped out of the vessel is drawn in through the pipe *B* to the end of the pipe *A* and is removed together with the water.

All that was said above about flows of liquid in pipes is applicable to a gas flow. If the velocity of a gas flow is not very high (lower than the velocity of sound in the gas), the gas is not compressed so that its volume changes and if, besides, we ignore friction, then Bernoulli's law is also valid for gas flows. In narrow sections of the pipes, where a gas moves faster, its pressure is lower than in wide sections and may become lower than the atmospheric pressure. In some cases, even a pipe is not required for this.

We can make a simple experiment. If we blow on a sheet of paper along its surface as is shown in Fig. 195, the paper will be lifted. This is due to a drop in pressure in the air jet over the paper.

The same phenomenon takes place in the flight of an aeroplane. A head-on air flow past the convex upper surface of the wing of the aeroplane causes a decrease in the pressure. The pressure over the wing turns out to be lower than the pressure under it (Fig. 196). It is due to this that a lifting force emerges. The theory of wings was developed by the outstanding Russian scientist N.E. ZHUKOVSKII, the "father of the Russian aviation", as Lenin called him.

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- ?"
- 1. Why is the velocity of a fluid higher in narrow parts of a pipeline than in wide sections?
- 2. What is the essence of Bernoulli's law?
- 3. Which force causes an increase in the velocity of a fluid, and hence in its kinetic energy as the fluid flows from a wide to a narrow section of a pipeline?
- 4. Can we assume that Bernoulli's law is a corollary of the energy conservation law?

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Fig. 195

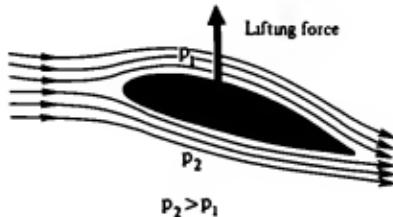


Fig. 196

## Summary

The work done by a force is a scalar quantity equal to the product of the magnitudes of the force and displacement by the cosine of the angle between their direction vectors. The work is positive if the angle is acute and negative if the angle is obtuse.

A force does work only if it is applied to a moving body. If several forces are applied to a *moving* body and their vector sum is zero (hence the body is in uniform motion), the algebraic sum of the works done by all the forces is zero while the work done by each of the forces differs from zero (except for the forces directed at right angles to the displacement).

If forces whose resultant differs from zero act on a body, the quantity  $mv^2/2$ , which characterizes the motion of the body and is called its kinetic energy, changes. Its change is equal to the work done by the resultant force.

If the force of gravity (in general, the force of universal gravitation) or an elastic force acts on a body, the change in the kinetic energy is accompanied by an equal and opposite change in the potential energy. If we are dealing with the force of gravity, the potential energy relative to the conventional zero level is  $mgh$ , where  $h$  is the height of the body above this level. In the case of an elastic force, the potential energy is  $kx^2/2$ .

When bodies interact through gravitational or elastic forces, the changes in kinetic and potential energy are equal and opposite. Therefore, for a closed system of interacting bodies, the law of conservation of total mechanical energy is observed.

If friction is also present in addition to elastic forces and forces of gravity, the total mechanical energy is not conserved. A certain fraction of this energy is transformed into the internal energy of the bodies between which friction exists.

## ON THE IMPORTANCE OF CONSERVATION LAWS

The laws of conservation of momentum and energy considered in the last two chapters have not only a deep physical meaning but also a philosophical significance. They indicate that the motion of matter can neither be exterminated nor created anew.

Indeed, when a moving body comes to a halt, it seems that the motion has vanished. When a body that has been at rest begins to move, it seems that a new motion which did not exist has been set in. However, the momentum and energy conservation laws indicate that this is not so. If a moving body has come to a halt, there is some reason behind that. It stopped under the action of some other body, i.e. due to the action of some force. If this is the force of friction, it means that the disappearance of the mechanical motion has led to the creation of another type of motion, viz. the motion of particles inside the body. If the body was made to stop by gravitational or elastic forces, then instead of one type of mechanical motion another type appears, viz. the motion of another body to which the body has transferred its momentum and energy, or the motion of the same body in the opposite direction.

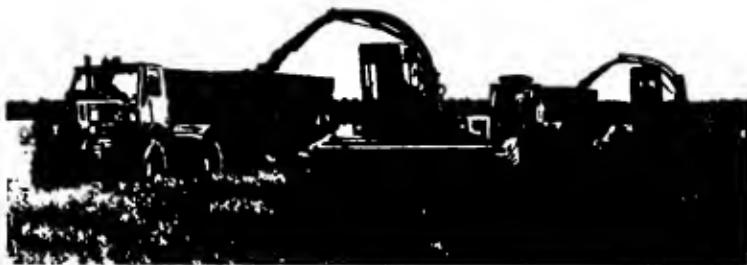
Thus, motion may change its form or be transferred from one body to another, but the laws of conservation of energy and momentum are observed during any change. There is no phenomenon or process in nature in which energy and momentum emerge or vanish without any compensation. This just means that the *motion of matter is conserved*.

It was shown above that the momentum and energy conservation laws make it possible to solve problems in mechanics when for some reason or other the forces acting on a body are unknown. But the importance of the laws of conservation is not limited to this. As far as is known at the moment, the energy and momentum conservation laws are absolutely accurate. This does not apply, for example, to Newton's second and third laws. When particles move at velocities close to the speed of light, Newton's laws are known to have a different form. In this sense, Newton's laws are approximate, while the energy and momentum conservation laws have no exceptions. If somebody declares that he has discovered a phenomenon or a process in which the conservation laws are violated, it can be said with confidence that the statement is erroneous.

The conservation laws serve as a guiding star while considering any problem associated with a natural phenomenon. It is a sort of a primary verification of the correctness of any statement. We shall often use the conservation laws in all branches of physics.

# CONCLUSION

Mechanics is an extensive science constituting one of the most important branches of physics which is an even more extensive science. We have discussed only some of its elements. Many sections of mechanics, such as the rotational motion of rigid bodies or vibrational motion, are beyond the scope of this course. Some problems in mechanics remain unsolved so far. Nevertheless, the course of mechanics presented in this book makes it possible to outline the typical features of this branch of science. To be more precise, we deal with the so-called classical, or Newtonian mechanics since it is based on Newton's laws. It has been shown that these laws are expressed in the form of mathematical relationships between a number of quantities



Harvesting of silage crop by harvester-shredders "Yaroslavets"

(momentum, acceleration, mass, force, and so on). The extent to which Newton's laws are exact remains to be determined.

**THE LIMITS OF APPLICABILITY OF NEWTONIAN MECHANICS.** Till the end of the 19th century, nobody doubted that these laws are perfectly correct. In the 20th century, however, it was found that Newton's laws are still not absolute. They cannot be used when bodies move at very high velocities comparable to the speed of light. A. EINSTEIN, who is called Newton of the 20th century, succeeded in formulating more general laws of motion which are valid for velocities close to the speed of light as well. These laws form the basis of the relativistic mechanics, or the theory of relativity.



Excavation for the railroad bed by dipper-excavator

Newton's laws are corollaries of these laws in the case when velocities of bodies are small in comparison with the speed of light

Newton's laws "fail" when the motion of intra-atomic particles is considered. There exists another "code of laws" quantum mechanics for the



Coal mining with the help of high-output wheel excavators



Earth-movers at work

motion of intra-atomic particles. Classical mechanics is also derived from it as a particular case. It is remarkable that the momentum and energy conservation laws derived from Newton's laws are valid both in quantum mechanics and in the theory of relativity.

Thus, mechanics forms the basis of natural science. We have considered only a small part of classical mechanics. However, we shall use it while studying the entire course of physics.

**MECHANICS AND MECHANIZATION.** Newton's laws were established when various machines and devices for replacing manual labour began to be used. The process of supplanting hard manual labour by appropriate mechanisms is continuing till our time. Mechanization has become a part of our life. Young people would, probably, not know that several decades ago construction workers carried bricks and other materials on their shoulders, climbing up the supporting structures.

Now this work is done by cranes mounted at each construction site. We know about the trade of barge haulers only from books and paintings, since this profession has vanished. The same is the case with the trade of stevedores, stokers, porters, etc. Their job is now done by appropriate mechanical devices.

We also know from literature about exhaustive labour of railroad builders. Nowadays, various machines such as excavators, bulldozers and track layers are used for constructing railroads.

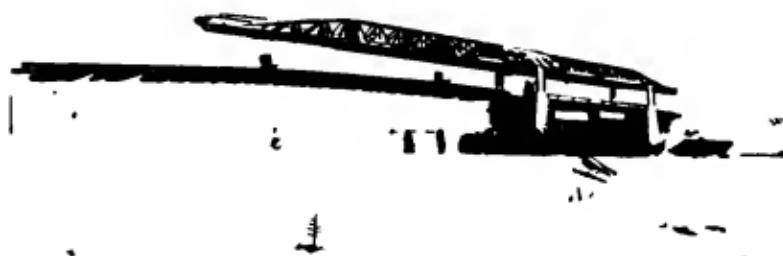
Of great importance are various agricultural machines that have transformed farming, one of the basic branches of economy. A special branch of industry deals with the production of such machines.

We are witnessing an industrial revolution in machine building itself. The work which was formerly done by many skilled workers is now done by automatic machines that do not involve manual labour.

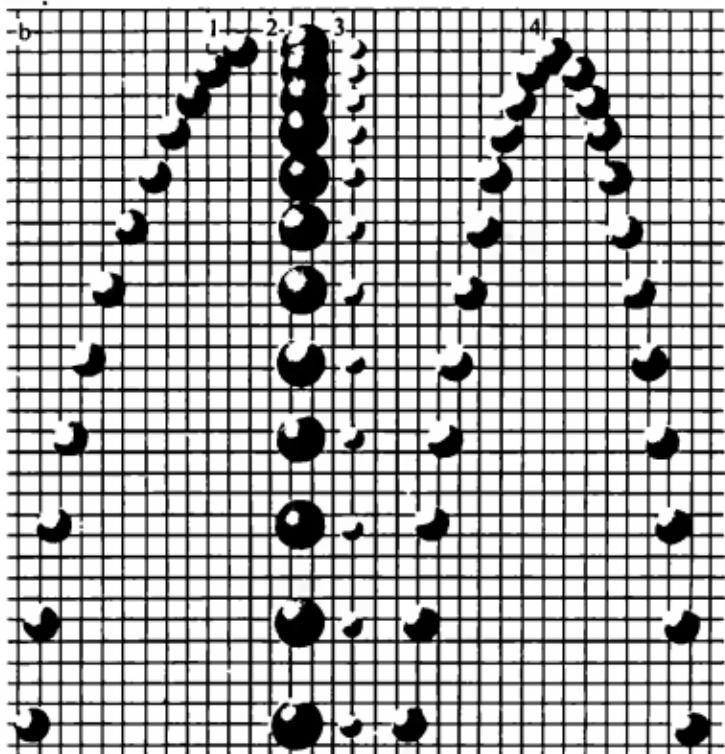


Gantry cranes are the helpmates of modern stevedores

All the variety of machines - from very simple to extremely complicated ones - are based on Newton's laws. The proper maintenance of these machines also requires the knowledge of these laws. There is no chance that these laws would "misfire". This explains their practical importance.



Railroad track laying on a subgrade ground work



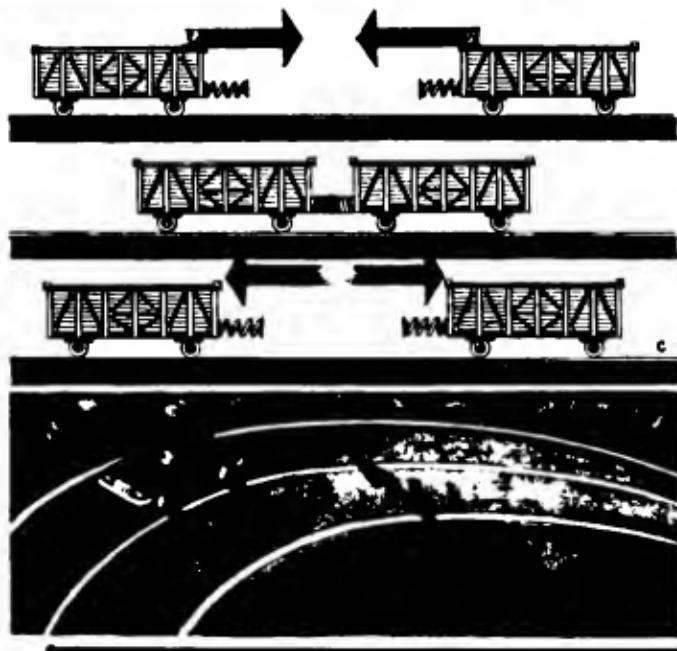
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(a) The path of a body thrown at an angle  $\alpha$  to the horizontal. If the air resistance were absent, the shell fired by a gun would fly in a parabola. The maximum range would correspond to an angle of  $45^\circ$  between the initial velocity and the horizontal.

At the angles of  $45^\circ - \alpha$  and  $45^\circ + \alpha$ , the range of the shell would be the same.

(b) The drawings are made from stroboscopic photographs of the motion of metallic balls under the action of the force of gravity: ball 1 is thrown along the horizontal;

balls 2 and 3 fall freely, while ball 4 is thrown at an angle to the horizontal.



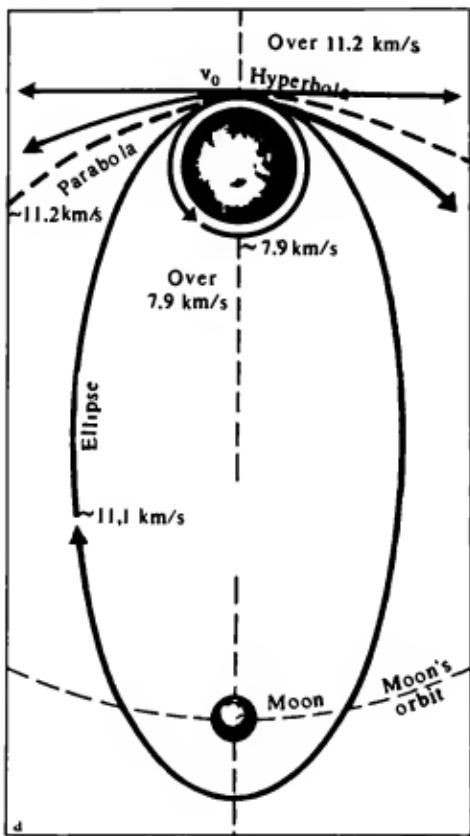
II

(a) An X-ray plate of a footballist's leg at the moment it kicks the ball. The deformations of the bones and of the ball are clearly seen. The elastic force acting on the ball emerges as a result of the deformation of the boot

(b) Drawn from the photograph of a racket and a tennis ball at the moment of impact.

(c) Elastic forces emerging during a collision of two bodies lead to a change in the velocities of the bodies

(d) A motorcar moves on the rounding of a highway with a centripetal acceleration. This acceleration is due to the friction force  $F_{fr}$  of the tyres on the surface of the road

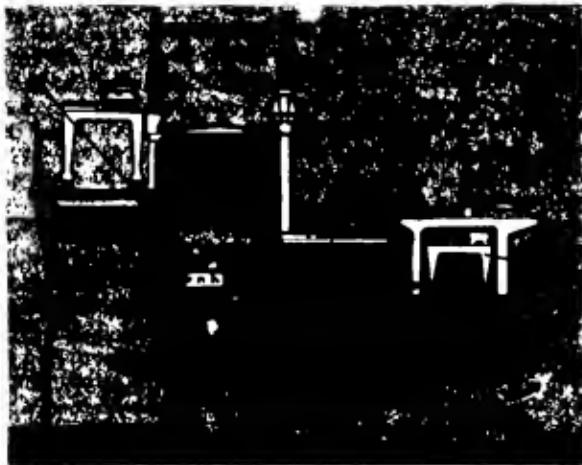
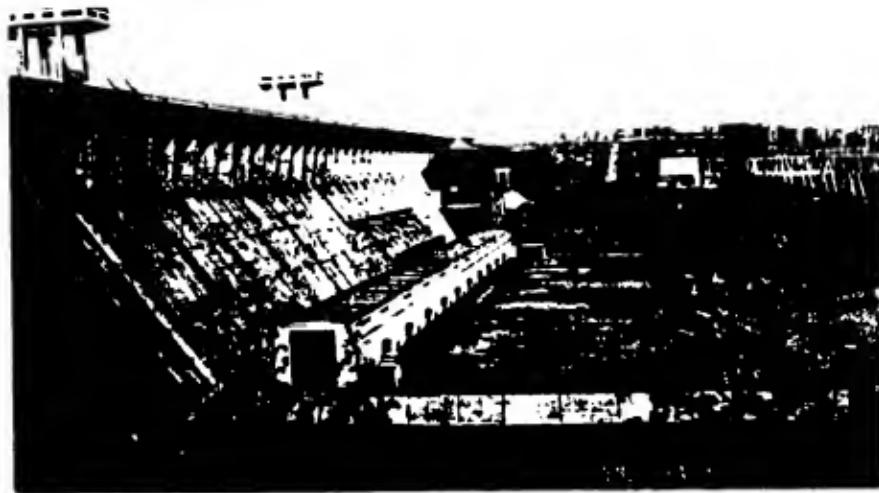


### III

(a) Space velocities. If the velocity  $v_0 \approx 7.9$  km/s of the spacecraft is parallel to the surface of the Earth, it becomes satellite of the Earth and moves in a circular orbit at a comparatively small height. If the velocity of the spacecraft is between 7.9 and 11.1 km/s, the orbit is elliptical. At a velocity of 11.2 km/s, the spacecraft moves in a parabola, and at a still higher velocity—in a hyperbola.

(b) Launching of a rocket





#### IV

Above, a photograph of a modern hydroelectric station. Below, a schematic diagram of a hydroelectric station. The potential energy of water is transformed into the kinetic energy as water falls from the upper to lower reach of the dam. As water flows through the turbine, its kinetic energy is transferred to the turbine-wheel and the generator coupled with it 1-turbine chamber, 2-hydroturbine, 3-water-wheel generator, 4-draught tube, 5-(electrical) switch-gears, 6-transformer, 7-gantry cranes

# PRACTICAL WORK<sup>1)</sup>

## 1.

### Determination of the Acceleration of a Body in Uniformly Accelerated Motion

**THE AIM OF THE WORK:** calculate the acceleration of a ball rolling down an inclined trough. For this purpose, the length of the displacement  $s$  of the ball is measured for a known time  $t$ . Since  $s = at^2/2$  for a uniformly accelerated motion with zero initial velocity, we can find the acceleration  $a = 2s/t^2$  of the ball by measuring  $s$  and  $t$ .

No measurement can be made with perfect accuracy. There is always a certain error due to the imperfection of measuring instruments and other reasons. However, even in spite of errors, there are several methods of making reliable measurements. The simplest of them is to calculate the arithmetic mean of the results of several independent measurements of the same quantity under constant experimental conditions. This method is used in this work.

**MEASURING INSTRUMENTS:** (1) a measuring tape, (2) a metronome.

**MATERIAL:** (1) a trough; (2) a ball; (3) a holder with clutches and a tang; (3) a metallic cylinder.

#### *Procedure*

1. Fix the trough with the help of the holder in an inclined position at a small angle to the horizontal (Fig. 197). Place the metallic cylinder into the trough at the lower end.

2. Releasing the ball (simultaneously with a metronome stroke) from the upper end of the trough, count the number of metronome strokes before the ball strikes the cylinder. It is convenient to carry out the experiment at 120 strokes per minute.

3. Changing the slope of the trough and slightly displacing the metallic cylinder, get four metronome strokes between the instant the ball is released and the instant it strikes the cylinder (three intervals between the strokes).

4. Calculate the time of motion of the ball.

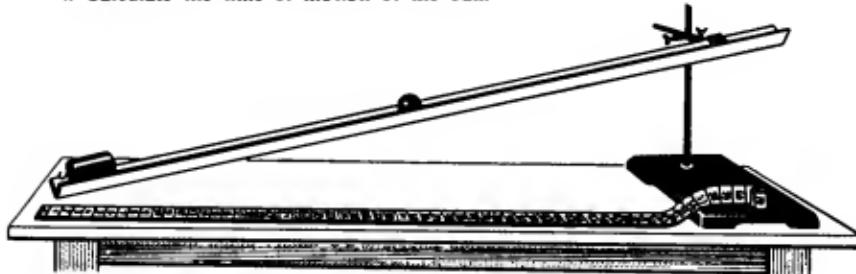


Fig. 197

<sup>1)</sup> The instructions to practical work were written with the assistance of Yu. I. Dik and G. G. Nikiphorov

5 Using the measuring tape, determine the displacement  $s$  of the ball. Without altering the slope of the trough (experimental conditions should remain unchanged), repeat the experiment five times, each time getting the coincidence between the fourth metronome stroke and the collision of the ball with the cylinder (the cylinder can be slightly shifted for this purpose).

6 Using the formula  $s_m = (s_1 + s_2 + s_3 + s_4 + s_5)/5$ , find the arithmetic mean of the magnitude of the displacement and then calculate the mean value of the magnitude of the acceleration:  $a_m = 2s_m/t^2$

7. Compile the results of measurements and calculations in tabular form

No. of experiment	$s, \text{ m}$	$s_m, \text{ m}$	Number of metronome strokes	$t, \text{ s}$	$a_m, \text{ m/s}^2$

## 2.

### Measurement of the Rigidity of a Spring

**THE AIM OF THE WORK:** find the rigidity of a spring by measuring elongation of the spring for different values of the external force, which balances the elastic force, using Hooke's law:  $k = F_{el}/|x|$ . In each experiment, the rigidity is determined at different values of the elastic force and elongation. In other words, experimental conditions are changed. Therefore, the mean value of the rigidity cannot be determined as the arithmetic mean of the results of several measurements. Let us employ the graphical method for determining the mean value, which can be applied in such cases. Using the results of several experiments, we plot the dependence of the elastic force  $F_{el} = mg$  on the magnitude of elongation  $|x|$ . While plotting the graph, we can find that the experimental points are not on the straight line corresponding to the formula  $F_{el} = k|x|$ . This is due to measurement errors. In this case, the graph should be drawn so that approximately the same number of points is on both sides of the straight line. After the graph has been plotted, a point is taken on the straight line (in the middle of the graph), the values of the elastic force and elongation corresponding to this point are determined, and the rigidity  $k$  is calculated. This will be just the required mean value  $k_m$  of the rigidity of the spring. The result of measurements is usually written as  $k = k_m \pm \Delta k$ , where  $\Delta k$  is the maximum absolute error in the measurement of rigidity. It is known from the introductory course of algebra that the relative error ( $\epsilon_k$ ) is the ratio of the absolute error ( $\Delta k$ ) to the value of  $k$ :  $\epsilon_k = \Delta k/k$ , whence  $\Delta k = \epsilon_k k$ . There exists a rule for calculating the relative error: if the quantity being determined in an experiment is found as a result of multiplication or division of approximate values appearing in computational formula, the relative errors are added. Here

$$\epsilon_k = \epsilon_m + \epsilon_g + \epsilon_r \quad (1)$$

**MEASURING INSTRUMENTS:** (1) set of weights with mass  $m_0 = 0.100 \text{ kg}$  each, the absolute error  $\Delta m_0 = 0.002 \text{ kg}$ , (2) millimetre ruler.

**MATERIAL:** (1) holder with clutches and tang; (2) spiral spring.

**Procedure**

1. Fix the end of the spiral spring on the holder (the other end of the spring is supplied with the pointer and the hook, Fig. 198).

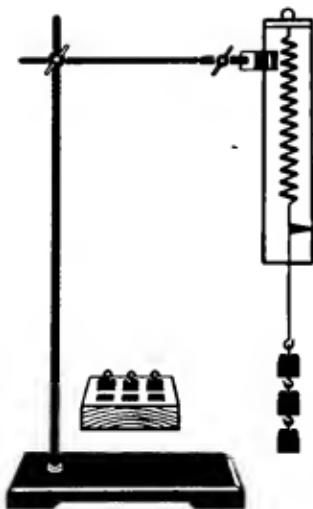


Fig. 198

2. Arrange and fix the millimetre ruler near the spring or behind it.
3. Mark and write down the division against which the pointer stands.
4. Suspend the load of a known mass from a spring and measure the spring elongation caused by it.
5. Add the second, third, etc. loads to the first one, each time writing down the elongation  $|x|$  of the spring. Write the results of measurements in the following table:

No. of experiment	$m, \text{ kg}$	$F_{el} = mg$ <sup>11) N</sup>	$ x , \text{ m}$

6. Using the results of measurements, plot the elastic force versus elongation. With the help of this graph, determine the mean value  $k_m$  of the rigidity of the spring.
7. Calculate the maximum relative error in determining the value of  $k_m$  (from the experiment with one load) In formula (1), we have

$$\epsilon_m = \frac{\Delta m}{m} = \frac{0.002 \text{ kg}}{0.1 \text{ kg}} = 0.02,$$

$$\epsilon_g = \frac{\Delta g}{g} = \frac{0.02 \text{ m/s}^2}{10 \text{ m/s}^2} = 0.02,$$

$$\epsilon_x = \frac{\Delta x}{x} = \frac{1 \text{ mm}}{25 \text{ mm}} = 0.04.$$

8. Find  $\Delta k = \epsilon_k k_m$  and write the answer in the form

$$k = k_m \pm \Delta k$$

<sup>11)</sup> Assume that  $g \approx 10 \text{ m/s}^2$ .

### 3.

## Determination of the Coefficient of Sliding Friction

**THE AIM OF THE WORK:** Using the formula  $F_{fr} = \mu P$  determine the coefficient of friction for a wooden block sliding over a wooden ruler. For this purpose, measure by a dynamometer the force which should be applied to the block with loads to make it move uniformly over the horizontal surface. This force is equal in magnitude to the friction  $F_{fr}$  acting on the block. Using the same dynamometer, the weight of the block with loads can be determined. This weight is equal to the normal pressure of the block on the surface over which it is sliding. Having determined in this way the values of friction for different values of the force of normal pressure, plot  $F_{fr}$  versus  $P$  and then find the mean value of the coefficient of friction (see work No. 2).

In this work, the dynamometer is the main measuring instrument. Its error  $\Delta g = 0.05 \text{ N}$  is equal to the measurement error if the pointer coincides with a scale division. If, however, the pointer does not coincide with a scale division in the measuring process (or oscillates), the measurement error for the force is  $\Delta F = 0.1 \text{ N}$ .

**MEASURING INSTRUMENTS:** dynamometer.

**MATERIAL:** (1) a wooden block; (2) a wooden ruler; (3) a set of loads.

#### Procedure

1. Put the block on the horizontal wooden ruler. Place the load on the block.
2. Having attached the dynamometer to the block, pull the latter along the ruler as uniformly as possible. Mark the reading of the dynamometer.
3. Weigh the block and the load.
4. Add the second and third loads to the first one, each time weighing the block and the loads and measuring the friction.

Using the results of measurement, fill in the following table:

No of experiment	$P, \text{ N}$	$\Delta P, \text{ N}$	$F_{fr}, \text{ N}$	$\Delta F_{fr}, \text{ N}$

5. From the results of measurement, plot the friction versus the force of pressure. Using this graph, find the mean value of the coefficient of friction  $\mu_m$  (see work No. 2).

6. Calculate the maximum relative error in the measurement of the friction coefficient. Since

$$\mu = F_{fr}/P, \epsilon_\mu = \epsilon_{F_{fr}} + \epsilon_P = \Delta F_{fr}/F_{fr} + \Delta P/P \quad (1)$$

(see work No. 2).

It follows from formula (1) that the largest error was made while measuring the coefficient of friction in the experiment with a single load (since in this case the denominators have the minimum value).

7. Find the absolute error  $\Delta\mu = \epsilon_\mu \mu_m$  and write the result in the form  $\mu = \mu_m \pm \Delta\mu$ .

## 4.

## Analysis of Motion of a Body Along a Parabola

THE AIM OF THE WORK measure the initial velocity imparted to a body in the horizontal direction, if the body subsequently moves under the force of gravity.

If a ball is thrown in the horizontal direction, it moves along a parabola. For the coordinate origin, we take the initial position of the ball. We direct the  $X$ -axis horizontally and the  $Y$ -axis, vertically downwards. Then at any instant  $t$ ,  $x = v_0 t$  and  $y = gt^2/2$ . The horizontal range  $l$  of the flight is the value of the coordinate  $x$  obtained by substituting for  $t$  the time in which the body falls from the height  $h$ . Hence we can write  $l = v_0 t$  and  $h = gt^2/2$ . Hence we can easily find the time  $t$  of falling and the initial velocity  $v_0$

$$t = \sqrt{\frac{2h}{g}} \quad \text{and} \quad v_0 = \frac{l}{t} \quad \text{or} \quad v_0 = l \sqrt{\frac{g}{2h}}$$

If the ball is thrown several times under the same experimental conditions, the values of the horizontal range will be spread to a certain extent due to various reasons whose effect cannot be taken into account. In such a case, the value of the quantity being measured is taken equal to the arithmetic mean of the results obtained in several experiments (see work No. 1).

MEASURING INSTRUMENTS. a millimetre ruler.

MATERIAL (1) a holder with clutches and a tang; (2) a chute for guiding the ball; (3) a plywood board; (4) a ball, (5) a sheet of paper, (6) drawing pins, (7) carbon paper.

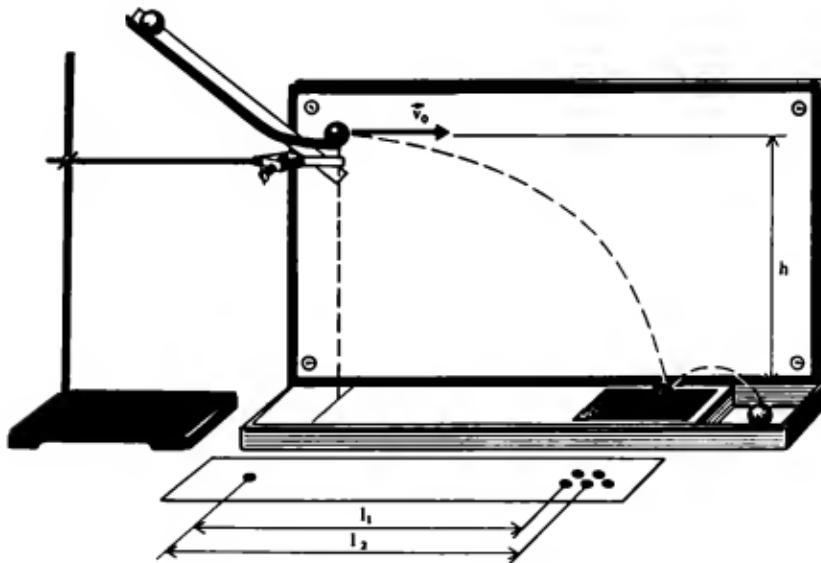


Fig. 199

#### Procedure

1. Using the holder, fix the plywood board in the vertical position. Fix the ridge of the chute with the same clutch. The bent end of the chute should be horizontal (Fig. 199).

2. Fix the sheet of paper whose width should not be less than 20 cm to the plywood by drawing pins. At the base of the set-up, put the carbon paper on a white paper strip.

3. Set the ball rolling five times from the same point of the chute and remove the carbon paper.

4. Measure the height  $h$  and the horizontal range  $l$  of the ball flight. Compile the results of measurement in tabular form.

5. Calculate the mean value of the initial velocity from the formula

$$v_{0m} = l_m \sqrt{\frac{g}{2h}}. \quad (1)$$

No of experiment	$h, \text{ m}$	$l, \text{ m}$	$l_m, \text{ m}$	$v_{0m}, \text{ m/s}$

6. Using the formulas  $x = v_m t$  and  $y = gt^2/2$ , find the coordinate  $x$  of the body (the  $y$ -coordinate has already been calculated) in every 0.05 s and plot the trajectory of motion on the sheet of paper pinned to the plywood board.

$t, \text{ s}$	0	0.05	0.10	0.15	0.20
$x, \text{ m}$	0				
$y, \text{ m}$	0	0.012	0.049	0.110	0.190

7. Let the ball move along the chute and make sure that its trajectory is close to the parabola plotted.

## 5. Analysis of Motion of a Body in a Circle

**THE AIM OF THE WORK:** prove that when a body moves in a circle under the action of several forces, their resultant is equal to the product of the mass of the body and its acceleration:  $\bar{F} = m\bar{a}$ . For this a conical pendulum is used (Fig. 200a). A body fixed to a thread is acted upon by the force of gravity  $\bar{F}_1$  and the elastic force  $\bar{F}_2$ . Their resultant  $\bar{F} = \bar{F}_1 + \bar{F}_2$ .

The force  $\bar{F}$  imparts to the body the centripetal acceleration

$$a = \frac{4\pi^2 r}{T^2}$$

( $r$  is the radius of the circle in which the load moves and  $T$  is the period of revolution).

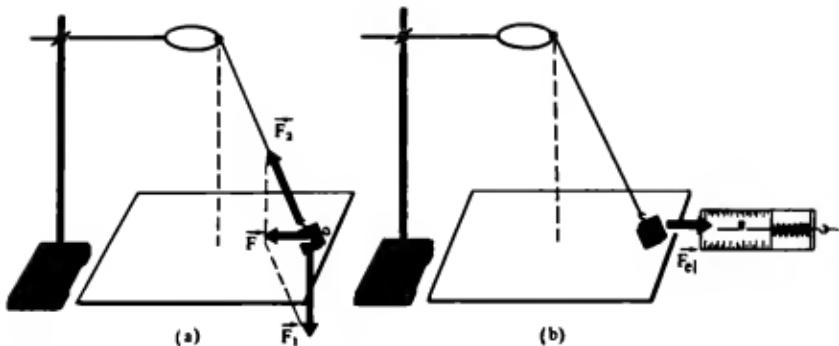


Fig. 200

According to Newton's second law,  $F = ma$ . Consequently,

$$F = m \frac{4\pi^2 r}{T^2}.$$

In order to find the period, it is convenient to measure the time  $t$  of a certain number  $N$  of revolutions. Then  $T = t/N$  and

$$F = m \frac{4\pi^2 N^2}{t^2} r. \quad (1)$$

The magnitude of the resultant  $F$  of the forces  $F_1$  and  $F_2$  can be measured by compensating it by the elastic force of the dynamometer spring as shown in Fig. 200b

In accordance with Newton's second law,  $F/m a = 1$ . Substituting into this formula the experimental values of  $F$ ,  $m$ ,  $N$ ,  $t_m$  and  $r$ , we may obtain a quotient differing from unity. This allows us to estimate the relative experimental error by the formula

$$\varepsilon = \frac{\left| \frac{F}{ma} - 1 \right|}{1}.$$

**MEASURING INSTRUMENTS:** (1) a millimeter ruler; (2) a timepiece with a second hand; (3) a dynamometer.

**MATERIAL:** (1) a holder with clutches and a tang; (2) a strong thread; (3) sheet of paper with a 15-cm circle drawn on it; (4) load.

#### Procedure

1. Attach the load to the thread about 45 cm long and suspend it from the ring of a holder.

2. One of the students must take the thread at the point of suspension by two fingers and set the pendulum in rotation.

3. The second student should measure with the tape the radius  $r$  of the circle described by the load (The circle can be drawn on the paper in advance and the load can be made move in this circle.)

4. Determine the period  $T$  of revolution of the pendulum with the help of a timepiece.

For this purpose, an ordinary watch with the second hand can be used. In this case, the student rotating the pendulum says aloud in phase with its turns: "zero,

zero", etc. The second student with the watch in his hand says "zero" when the second hand is in a position convenient to be used as the reference point. After this, the first student counts aloud the number of turns. Having counted 30-40 turns ( $N$ ), the time interval  $t$  is fixed. The experiment is repeated five times.

5. Calculate the mean value of the force from formula (1), taking into account that  $\pi^2 = 10$  with the relative error below 0.015.

6. Measure the magnitude of the resultant  $\vec{F}$  by balancing it with the elastic force of the dynamometer spring (see Fig. 200b).

7. Compile the results of measurement in a tabular form.

No. of experiment	$t$ , s	$t_m$ , s	$N$	$m$ , kg	$r$ , m	$ma = \frac{4\pi^2 N^2}{t_m^2} r$ , N	$F$ , N

8. Compare the ratio  $F/ma$  with unity and draw the conclusion about the error in the experimental verification of the fact that the centripetal acceleration is imparted to a body by the vector sum of the forces acting on it.

## 6. Equilibrium Conditions for a Lever

THE AIM OF THE WORK: establish the relation between the moments of forces applied to the arms of a lever in equilibrium. For this purpose, one or several loads are suspended from the lever arm, and to the other arm, the dynamometer is attached (Fig. 201). Using this dynamometer, we can measure the magnitude of the force  $F$  required for the lever to be in equilibrium. Then, using the same dynamometer, the magnitudes of the weight  $P$  of the loads are measured. The lengths of the lever arms are measured with a ruler. After this, the magnitudes of the moments  $M_1$  and  $M_2$  of the forces  $P$  and  $F$  are determined:

$$M_1 = Pl_1 \quad \text{and} \quad M_2 = Fl_2.$$

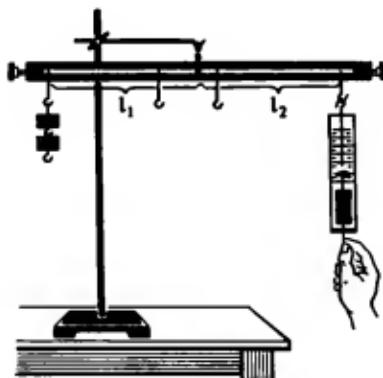


Fig. 201

We can draw a conclusion about the error in the experimental verification of the rule of moments by comparing the ratio  $M_1/M_2$  with unity (see work No 5).

**MEASURING INSTRUMENTS:** (1) a ruler; (2) a dynamometer,  $\Delta g = 0.05$ .

**MATERIAL:** (1) a holder with a clutch; (2) a lever; (3) a set of loads.

**Procedure**

1. Fix the lever on the holder and balance it in the horizontal position with the help of movable nuts at its ends.
2. Suspend a load from a certain point of a lever arm.
3. Attach the dynamometer to the other arm of the lever and find the force which should be applied so that the lever is in equilibrium.
4. Measure the lengths of the lever arms with the help of the ruler.
5. Measure the weight  $P$  of the load with the help of the dynamometer.
6. Find the magnitudes of the moments of the forces  $P$  and  $F$ .
7. Compile the obtained values in a tabular form:

$l_1, \text{ m}$	$l_2, \text{ m}$	$P, \text{ N}$	$F, \text{ N}$	$M_1 = Pl_1, \text{ N m}$	$M_2 = Fl_2, \text{ N m}$

8. Compare the ratio  $M_1/M_2$  with unity and draw the conclusion about the relative error of the experimental verification of the rule of moments

## 7.

### Determination of the Centre of Gravity of a Flat Plate

**THE AIM OF THE WORK:** find the centre of gravity of the plate.

If a flat plate is suspended from some point, it turns so that the vertical drawn through the point of suspension (Fig. 202) passes through the centre of gravity of the

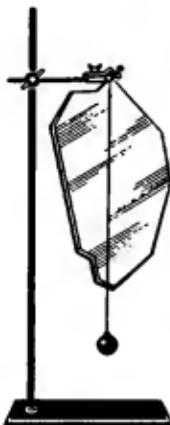


Fig. 202

plate. This allows us to find the centre of gravity of a plane plate experimentally. For this purpose, we must suspend a plate from a certain point and draw the vertical through the point of suspension. Then the experiment must be repeated for another point of suspension. The point of intersection of the straight lines drawn on the plate is the centre of gravity of the plate.

In order to make sure that this is so, we must suspend the plate from a third point. The vertical passing through the point of suspension must pass through the point of intersection of the first two straight lines.

We can also balance the plate on the pin. The plate is in equilibrium if the point of support coincides with the centre of gravity.

**MATERIAL:** (1) a ruler; (2) a flat plate of arbitrary shape; (3) a plumb-line; (4) a pin; (5) a holder with a tang and a clutch; (6) a cork.

*Procedure*

1. Fix the cork in the holder in the horizontal position.
2. Using the pin suspend the plate and the plumb-line.
3. Mark the vertical with a sharp pencil on the lower and upper edges of the plate.
4. Take the plate off the holder and draw the line connecting the marking points.
5. Repeat the experiment, suspending the plate from another point.
6. Make sure that the point of intersection of the straight lines drawn on the plate is its centre of mass.

## 8.

### Experimental Investigation of the Law of Conservation of Mechanical Energy

**THE AIM OF THE WORK:** compare two quantities, viz. the decrease in the potential energy of a body attached to a spring during its fall and the increase in the potential energy of the stretched spring.

**MEASURING INSTRUMENTS:** (1) a dynamometer whose spring has a rigidity of  $(40 \pm 4)$  N/m; (2) a measuring ruler; (3) a load; the mass of the load is  $(0.100 \pm 0.002)$  kg.

**MATERIAL:** (1) an index pin; (2) a holder with a clutch and a tang.

Use the set-up shown in Fig. 203. It consists of a dynamometer with the index pin 1, fixed on the holder. The dynamometer spring has a wire rod with a hook at its end. The index pin (it is shown separately on a magnified scale and marked by 2) is a light cork plate ( $5 \times 7 \times 1.5$  mm in size) cut with a knife up to its centre. It is pinned onto the wire rod of the dynamometer. The index pin must slide along the rod with a small friction, which is, however, sufficient for balancing the force of gravity acting on the cork plate. This should be checked before starting. For this purpose, the index pin is placed at the lower end of the scale on the limiting clamp. Then the spring is stretched and released.

The index pin must go up together with the wire rod, thus marking the maximum elongation of the spring, which is equal to the distance between the prop and the index pin.

If the load suspended from the dynamometer hook is lifted so that the spring is not stretched, the potential energy of the load relative to, for example, the surface of the table is  $mgh$ . When the load falls (is lowered by a distance  $x = h$ ), its potential energy decreases by  $E_1 = mgh$ , while the potential energy of the spring increases upon deformation by  $E_2 = kx^2/2$ .

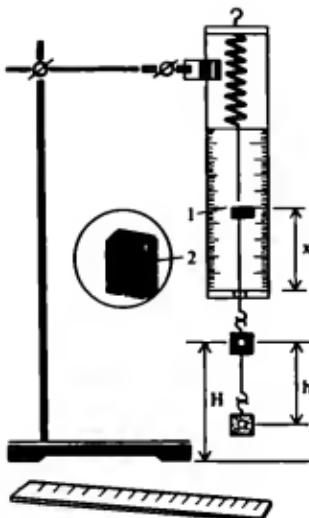


Fig. 203

*Procedure*

1. Fix the load tightly on the dynamometer hook.
2. Lift the load by the hand, unloading the spring, and place the index pin in the lower position at the prop.
3. Release the load. While falling down, it stretches the spring. Take the load off and measure by the ruler the maximum elongation  $x$  of the spring using the position of the index pin.
4. Repeat the experiment five times.
5. Calculate  $E_{1m} = mgh_m$  and  $E_{2m} = kx_m^2/2$
6. Compile the results in a tabular form:

No. of experiment	$x_{max}, \text{ m}$	$x_m = h_m$	$E_{1m}, \text{ J}$	$E_{2m}, \text{ J}$	$E_{1m}/E_{2m}$

7. Compare the ratio  $E_{1m}/E_{2m}$  with unity and estimate the error with which the law of energy conservation was verified.

## Answers to Exercises

Ex. 1. 1.  $s_x = 4 \text{ m}$ ,  $s_y = -3 \text{ m}$ . 2.  $x = 2.2 \text{ m}$ ,  $y = 4 \text{ m}$ ,  $\sim 6 \text{ m}$ . 3. 13 km.

Ex. 2. 1.  $\sim 3.5 \text{ km}$  South-East, 42 min. 2. 90 km/hr. 3. 3.4 km.

Ex. 3. 1. 7.5 m. 2. 0.1 m/s.

Ex. 4. 1. 950 km/hr, 850 km/hr. 2. 225 s.

3.  $x = 72 \text{ km}$ ,  $y = 1440 \text{ km}$ ,  $z = 8 \text{ km}$ . The  $X$ -axis is directed from West to East, the  $Y$ -axis, from South to North, and the  $Z$ -axis vertically upwards. 4. No. Yes, the time required to cross the river along the shortest path is longer than in still water.

Ex. 5. 1. 70 km/hr. 2.  $\sim 55 \text{ km/hr}$ .

Ex. 6. 1. 10 s. 2.  $2.5 \text{ m/s}^2$ . 3.  $\sim 6.3 \text{ s}$ . 4.  $64800 \text{ km/hr}$ .

Ex. 7. 1. 27 m, 4 s, 8 m. 2. The first body moves uniformly while the second and third bodies are in uniformly accelerated motion. At the moment corresponding to point  $A$  we have:  $v_{1x} = v_{2x} = 2 \text{ m/s}$ ,  $v_{3x} = 0.5 \text{ m/s}$ , at point  $B$ :  $v_{1x} = v_{3x} = 2 \text{ m/s}$ ,  $v_{2x} = 8 \text{ m/s}$ ,  $a_{1x} = 0$ ,  $a_{2x} = 2 \text{ m/s}^2$ ,  $a_{3x} = 0.5 \text{ m/s}^2$ . 3. (a)  $a_{1x} = 1 \text{ m/s}^2$ ,  $a_{2x} = 0.4 \text{ m/s}^2$ ,  $a_{3x} = -0.5 \text{ m/s}^2$ . 4. (a)  $OA = 9 \text{ m/s}$ ,  $OB = 3 \text{ m/s}$ ,  $OC = 4.5 \text{ m/s}$ . (b)  $a_{1x} = a_{2x} = 1 \text{ m/s}^2$ ,  $a_{3x} = -2 \text{ m/s}^2$ . 5.  $\sim 6.7 \text{ m/s}^2$ , 750 m. 6. 0.6 m. 7. 2.4 km. 8.  $1.6 \times 10^4 \text{ km}$ . 9.  $\sim 3.8 \text{ m/s}^2$ . 10. 500 m. 11.  $\sim 700 \text{ m}$ .

Ex. 8. 1.  $\sim 3.1 \text{ m/s}$ . 2.  $\sim 2.3 \text{ m/s}^2$ . 3.  $\sim 7.8 \text{ km/s}$ . 4.  $\sim 19 \text{ m/s}$ . 5.  $\sim 2.7 \times 10^{-3} \text{ m/s}^2$ .

Ex. 9. 1. 6 m/s. 2. 2 cm, 6 cm. 3. 12 cm.

Ex. 10. 1. 1. 2. 30 cm/s.

Ex. 11. 1. 9.8 N 2. 4 kN. 3. 2400 N. 4. The statement is false: the time is shorter by a factor of  $\sqrt{2}$  and not 2.

Ex. 12. 1. No. 2.  $0.25 \text{ m/s}^2$ ,  $0.2 \text{ m/s}^2$ .

Ex. 13. 1. 49 N/m. 2. 8 cm.

Ex. 14. 1. 0.7 mg. 2.  $\sim 0.17 \text{ N}$ . 3.  $\sim 2 \times 10^{20} \text{ N}$ . 4.  $\sim 560$  times.

Ex. 15. 1. 5 kg. 2.  $\sim 2600 \text{ km}$ . 3.  $\sim 1.6 \text{ N}$ ,  $\sim 6.3$  times smaller. 4.  $\sim 3.7 \text{ m/s}^2$ .

Ex. 16. 1. 49 N. 2.  $\sim 1100 \text{ kg}$ . 3. 75 N.

Ex. 17. 1.  $\sim 78 \text{ m}$ . 2.  $\sim 10.5 \text{ s}$ ,  $\sim 103 \text{ m/s}$ . 3. 1 s, 9.8 m/s. 4.  $\sim 11 \text{ m/s}$ . 5.  $\sim 20 \text{ m/s}$ , 15 m/s. 7.  $\sim 46 \text{ m}$ . 8.  $\sim 78 \text{ m}$ ,  $\sim 39 \text{ m/s}$ . 9.  $\sim 3.3 \text{ m}$ , 8.1 m/s,  $\sim 1.3 \text{ s}$ ,  $\sim 0.8 \text{ m}$ . 10. 75 m, 10 m/s, 10 m/s. 11. Two times. 12. 12 m/s.

Ex. 18. 1. 1.3 m,  $\sim 1.0 \text{ s}$ ,  $\sim 8.8 \text{ m}$ . 2.  $\sim 2.8 \text{ m}$ .

Ex. 19. 1. 4900 N in all cases. 2. (a) 1010 N, (b) 980 N, (c) 940 N, (d) 0. 3. It will decrease by 5600 N. 4. 9.8 N,  $\sim 9.77 \text{ N}$ .

Ex. 20. 1.  $\sim 90 \text{ min}$ . 2. 5.59 km/s. 3. 4700 km. 4. 36 000 km.

Ex. 21. 1.  $\sim 10 \text{ m/s}$ . 2.  $\sim 3.4 \text{ s}$ ,  $\sim 34 \text{ m}$ .

Ex. 22. 2.  $\sim 2 \text{ m/s}$ . 3.  $30^\circ$ . 4.  $\sim 10 \text{ m/s}^2$ . 5.  $\sim 5.5 \text{ m/s}^2$ . 6.  $\sim 16 \text{ N}$ .

Ex. 23. 1. No,  $\sim 50 \text{ km/hr}$ . 2.  $\sim 71 \text{ km/hr}$ .

Ex. 24. 1. 866 N, 1000 N,  $\sim 707 \text{ N}$ , 500 N. 2.  $\sim 30 \text{ N}$ , 7.9 N. 3.  $\sim 3400 \text{ N}$ .

Ex. 25. 1. 0.1 kg. 2. 0.2 kg. 3. Yes. 4.  $\sim 1730 \text{ N}$ , 2000 N.

Ex. 26. 1.  $10 \text{ kg} \cdot \text{m/s}$ . 2. (a)  $3 \times 10^4 \text{ kg} \cdot \text{m/s}$ , (b)  $6 \times 10^4 \text{ kg} \cdot \text{m/s}$ . 3. 0.2 kg m/s, 2 N 4.  $\sim 20000 \text{ kg} \cdot \text{m/s}$ , 1000 kg. 5.  $\sim 3.4 \text{ s}$ .

Ex. 27. 1. 5.5 m/s. 2. 0.3 m/s. 3. 4.5 kg.

Ex. 28. 1. 500 J.  $\sim$  0.66. 2.  $\sim$   $-1.1 \times 10^3$  J.

Ex. 29. 1. 180 J.  $\sim$  11 m/s. 2.  $\sim 4.5 \times 10^8$  J. 3.  $\sim 4.0 \times 10^{10}$  J. 4. 40 N, along the radius,  $A = 0$ . 5.  $\sim 200\,000$  J, 1000 kg. 6.  $\sim 34$  m.

Ex. 30. 1.  $\sim 120$  J. 2.  $\sim -1.1 \times 10^{-4}$  J. 3.  $2.7 \times 10^5$  J,  $\sim 1.6 \times 10^6$  J.

Ex. 31. 1. 8 J. 2.  $\sim 16$  J. 3. 0.085 J. 4. Elongations differ in sign, while the potential energies are equal. 5.  $\sim 0.02$  J. 6. 8 J.

Ex. 32. 1.  $\sim 46$  m. 2.  $\sim 2000$  m. 3.  $\sim 290$  J,  $\sim 590$  J. 4.  $\sim 230$  kg. 5.  $\sim 0.01$  m. 6.  $\sim 1.6$  m/s. 7. 3.75 m/s, 6.25 m/s

Ex. 33. 1.  $\sim -240$  J. 2.  $\sim 2.7 \times 10^3$  J. 3. 36 km/hr. 4. The kinetic energy decreased by 1500 J. 5.  $\sim 700$  kJ. 6. It moved in air. 7.  $\sim 1800$  J.

Ex. 34. 1. 7200 N. 2. 8 t. 3. 360 kJ. 4.  $\sim 7.9 \times 10^{13}$  J. 5. 20 kW.

Ex. 35. 1. 6 t. 2.  $\sim 5700$  N. 3. 77 t. 4.  $\sim 20\%$ .

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## Derived SI Units

Quantity	Units
Velocity	metre per second
Angular velocity	radian per second
Acceleration	metre per second squared
Force	newton
Moment	newton-metre
Rigidity	newton per millimetre
Momentum	kilogram-metre per second
Impulse	newton-metre
Work, energy	joule
Power	watt

## Largest and Smallest

### Distance

$10^{26}$  m is the distance from the most remote object to the Earth that can be observed with a modern telescope

Notation	
second	$\text{m/s}$
r second	$\text{rad/s}$
second	$\text{m/s}^2$
	$\text{N}$
metre	$\text{N}\cdot\text{m}$
per metre	$\text{N/m}$
-metre per second	$\text{kg}\cdot\text{m/s}$
per second	$\text{N/s}$
	$\text{J}$
	$\text{W}$

## smallest Quantities

### Velocity

$3 \times 10^8 \text{ m/s}$  is the maximum possible velocity  
and is the speed of light in vacuum

## Acceleration

$10^{23} \text{ m/s}^2$  is the acceleration experienced by an electron in a hydrogen atom

## Time

$10^{18} \text{ s}$  is the age of the Earth

$10^{-24} \text{ s}$  is the time it takes light to travel the diameter of a hydrogen nucleus

## Mass

$10^{30} \text{ kg}$  is the mass of the Sun

$10^{-30} \text{ kg}$  is the mass of an electron



## Density

$4 \times 10^{17} \text{ kg/m}^3$  is the density of nuclear matter

$10^{-21} \text{ kg/m}^3$  is the average density of interstellar matter

## Power

$6 \times 10^6 \text{ kW}$  is the power of the Krasnoyarsk hydroelectric plant which is one of the largest in the world